MUMBAI UNIVERSITY

SEMESTER-1

ENGINEERING MECHANICS SOLVED PAPER-DECEMBER 2016

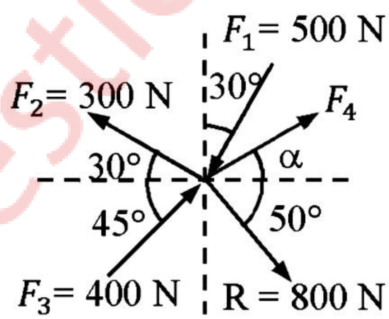
N.B:-(1)Question no.1 is compulsory.

(2)Attempt any 3 questions from remaining five questions.

(3)Assume suitable data if necessary,and mention the same clearly.

(4)Take g=9.81 m/s^2,unless otherwise specified.

Q.1(a) Find the force F_4, so as to give the resultant of the force as shown in the figure given below. (4 marks)

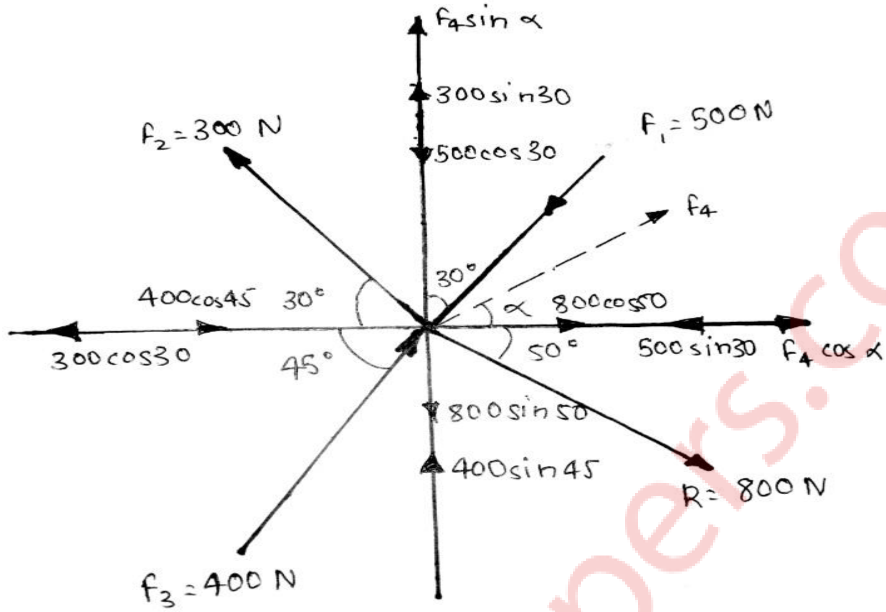


Solution :

Given : Forces and their resultant

To find : Force F_4

Solution :



Assume that force F_4 acts at an angle θ

Taking forces having direction towards right as positive and forces having direction upwards as positive.

Resolving forces along X direction :

$-F_1\sin30 - F_2\cos30 + F_3\cos45 + F_4\cos\theta = R\cos50$

$-500\sin30 - 300\cos30 + 400\cos45 + F_4\cos\theta = 800\cos50$

$F_4\cos\theta = 741.195$ …………(1)

Resolving forces along Y direction :

$-F_1\cos30 + F_2\sin30 + F_3\sin45 + F_4\sin\theta = -R\sin50$

$-500\cos30 + 300\sin30 + 400\sin45 + F_4\sin\theta = -800\sin50$

$F_4\sin\theta = -612.6656$ ……….(2)

Squaring and adding (1) and (2)

$(F_4\sin\theta)^2 + (F_4\cos\theta)^2 = (-612.6656)^2 + (741.195)^2$

$F_4^2(\sin^2\theta + \cos^2\theta) = 924729.1173$

$F_4 = 961.6284$ N

Dividing (2) by (1)

$\frac{F_4 sin\theta}{F_4 cos\theta} = \frac{-612.6656}{741.195}$

tan θ = -0.8266

θ = 39.5769° (in fourth quadrant)

F_4 = 961.6284 N (at an angle 39.5769° in fourth quadrant)

Q.1(b) A particle starts from rest from origin and it's acceleration is given by a = $\frac{k}{(x+4)^2}$ m/s². Knowing that v = 4 m/s when x = 8m,find :

(1)Value of k

(2)Position when v = 4.5 m/s (4 marks)

Solution :

Given : Particle starts from rest

$a = \frac{k}{(x+4)^2}$ m/s²

v = 4m/s at x = 8m

To find : Value of k and position when v= 4.5m/s

Solution:

We know that $a = v\frac{dv}{dx}$

$v\frac{dv}{dx} = \frac{k}{(x+4)^2}$

$v\,dv = k(x+4)^{-2}\,dx$

Integrating both sides

$\int vdv = \int k(x+4)^{-2}\,dx$

$\frac{v^2}{2} = \frac{-k}{x+4} + c_1$ ………(1)

Putting x=0 and v=0

$c_1 = \frac{k}{4}$ ……….(2)

$\frac{v^2}{2} = \frac{-k}{x+4} + \frac{k}{4}$ …….(From 1 and 2) ……..(3)

k = 48

From (3)

$\frac{v^2}{2} = \frac{-48}{x+4} + \frac{48}{4}$

$v^2 = 24 - \frac{96}{x+4}$

Substituting v=4.5 m/s

$4.5^2 = 24 - \frac{96}{x+4}$

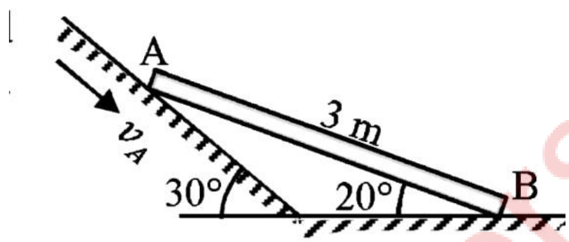
$\frac{96}{3.75} = x+4$

x = 21.6 m

Value of k = 48

The particle is at a distance of 21.6 m from origin when v = 4.5 m/s

Q.1(c)Rod AB of length 3 m is kept on a smooth plane as shown in the given figure.The velocity of end A is 5 m/s along the inclined plane.Locate the ICR and find velocity of end B. (4 marks)



Solution :

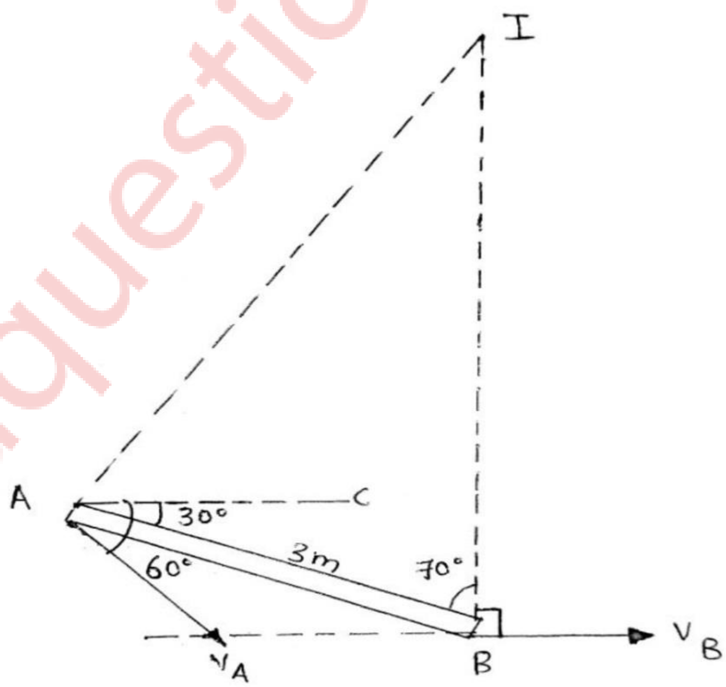
Given : Length of rod AB = 3m

v_a = 5 m/s

To find: ICR

Velocity of end B

Solution :



Solution:

Given : AB=3m

v_A=5m/s

To find : ICR

v_B

Solution:

ICR is shown in the diagram denoted by point I

Assume ω to be the angular velocity of rod AB

BY GEOMETRY:

∠CAD=30°, ∠ABD=20°

∠CAB= ∠ABD=20°

∠CAI=90°-30°

=60°

∠BAI = ∠CAI+ ∠CAB=60°+20°

=80°

In △IAB. ∠AIB = 180°-80°-70°

=30°

BY SINE RULE :

$$\frac{AB}{\sin I} = \frac{IB}{\sin A} = \frac{IA}{\sin B}$$

$$\frac{3}{\sin 30} = \frac{IB}{\sin 80} = \frac{IA}{\sin 70}$$

IB=5.9088 m

IA=5.6382 m

$\omega = \frac{v_a}{r} = \frac{v_a}{IA} = \frac{5}{5.6382}$ = 0.8868 rad/s(anti-clockwise)

v_B = r ω

= IB x ω

= 5.9088 x 0.8868

=5.2401 m/s (Towards right)

Velocity of end B=5.2401 m/s(towards right)

Q.1(d)What is a zero force member in a truss? With examples state the conditions for a zero force member. (4 marks)

Solution:

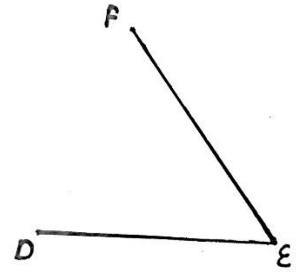
1. In engineering mechanics, a **zero force member** is a **member** (a single truss segment) in a truss which, given a specific load, is at rest that is it is neither in tension, nor in compression

2.The conditions for a zero force member are :

(a) In a truss,at a joint there are only three members of which two are collinear and if the joint has no external load then the non collinear members is a zero force member.

e.g.: DF is a zero force member in the given figure.

(b) In a truss,if at an unsupported joint there are only two members and if the joint has no external load then both the members are zero force members.



e.g.:DE and EF are zero force members in the given figure.

Q1(e)A glass ball is dropped on a smooth horizontal floor from which it bounces to a height of 9m.On the second bounce it rises to a height of 6 m.From what height was the ball dropped and find the coefficient of restitution between the glass and the floor. (4 marks)

Solution:

Given : First bounce height = 9 m

Second bounce height = 6 m

To find : Co-efficient of restitution

Solution :

Assume the ball fall from height h and then rebounds to height h_1

Before first bounce :

u = 0 , s = h , a = -g

Velocity after first bounce

$u_1 = ev = e\sqrt{2gh}$ ………(e is the co-efficient of restitution)

Using kinematical equation : $v_1^2 = u_1^2 + 2as_1$

$0^2 = e^2 \times 2gh - 2gh_1$

$2gh_1 = e^2 \times 2gh$

$h_1 = e^2h$ ……….(1)

Assume the ball rises to height of h_2 after the second bounce

$h_2 = e^2h_1$ …….(2)

Putting h_1 = 9 m and h_2 = 6 m

$6 = e^2 \times 9$

$e^2 = \frac{6}{9}$ ……..(3)

e = 0.8165

From (1) and (3)

$9 = \frac{6}{9} \times h$

h = 13.5 m

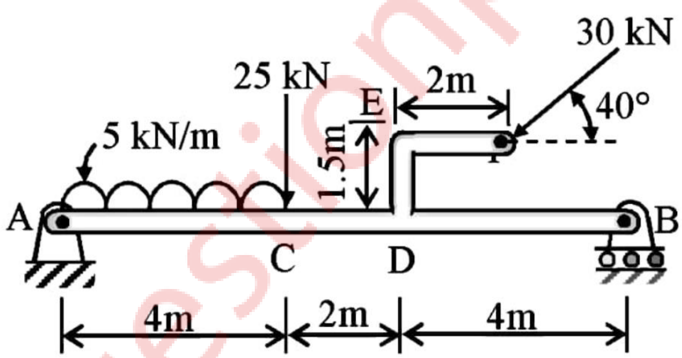
Co-efficient of restitution = 0.8165

Height from which ball was dropped = 13.5 m

Q2(a)The given figure shows a beam AB hinged at A and roller supported at B.

The L shaped portion is welded at D to the beam AB.

For the loading shown,find the support reactions. (8 marks)

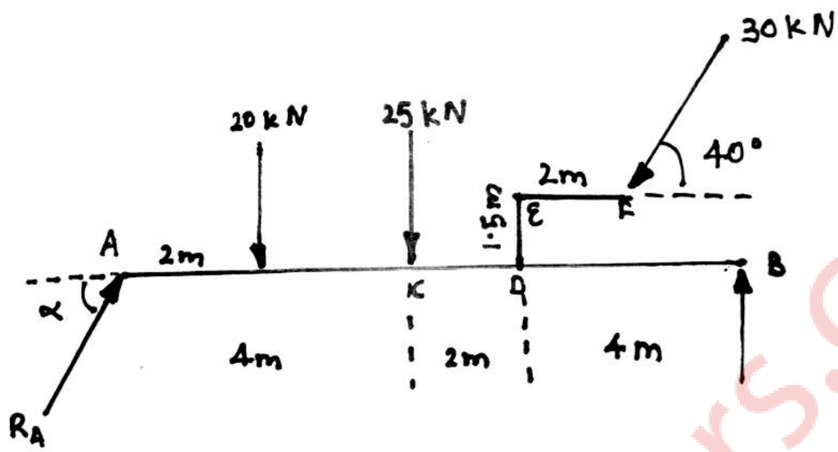


Solution :

Given : Beam AB hinged at A and roller supported at B and different forces acting on it.

To find : Support reactions

Solution :



Force of distributed load AC = 5 x 4

= 20 kN

Distance of force acting from point A = $\frac{4}{2}$ =2m

The beam is in equilibrium

Applying the conditions of equilibrium

$\Sigma M_A = 0$

$-20 \times 2 - 25 \times 4 - 30\sin40 \times 8 + 30\cos40 \times 1.5 + R_B \times 10 = 0$

$10R_B = 40+100+240\sin40-45\cos40$

$10R_B = 259.797$ kN

R_B = 25.9797 kN (Acting upwards)

Applying the conditions of equilibrium

$\Sigma F_X = 0$

$R_{AX} - 30\cos40 = 0$

$R_{AX} = 22.9813$ kN ………(1)

$\Sigma F_Y = 0$

$R_{AY} - 20 - 25 - 30\sin40 + R_B = 0$

$R_{AY} = 38.3039$ kN ………..(2)

$R_A = \sqrt{R_{AX}^2 + R_{AY}^2}$

$R_A = \sqrt{22.9813^2 + 38.3039^2}$

R_A = 44.6691 kN

$\alpha = \tan^{-1}\left(\frac{R_{AY}}{R_{AX}}\right)$

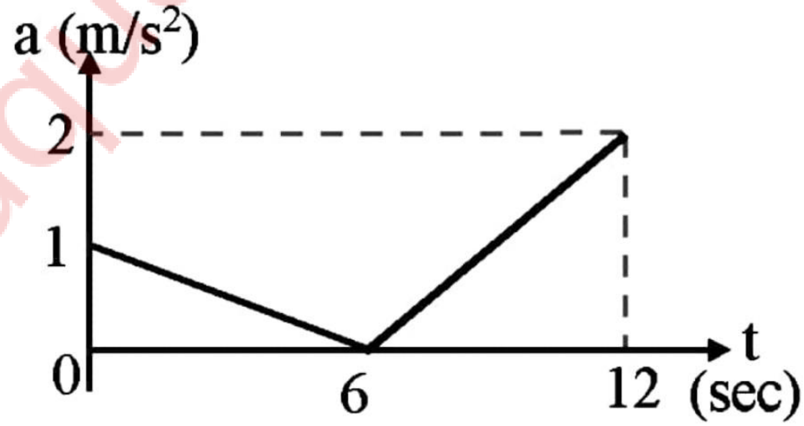
$= \tan^{-1}\frac{38.3039}{22.9813}$

$\alpha = 59.0374^o$

Reaction at hinge A = 44.6691 kN (59.0374° in first quadrant)
Reaction at roller B = 25.9797 kN (Towards up)

Q.2(b)The acceleration time diagram for a linear motion is shown.

Construct velocity time diagram and displacement time diagram for the motion assuming that the motion starts with a initial velocity of 5 m/s from the starting point. (6 marks)



Solution :

Given : Acceleration time graph

To draw : Velocity time graph

Displacement time graph

Solution :

FOR VELOCITY TIME GRAPH :

We know that the area under a-t graph gives the velocity.

AB on a-t graph represents linearly varying deceleration

$v_0 = 5$ m/s

$v_1 = v_0 + A(\triangle\ OAB)$

$= 5 + \frac{1}{2} \times 6 \times 1$

$= 8$ m/s

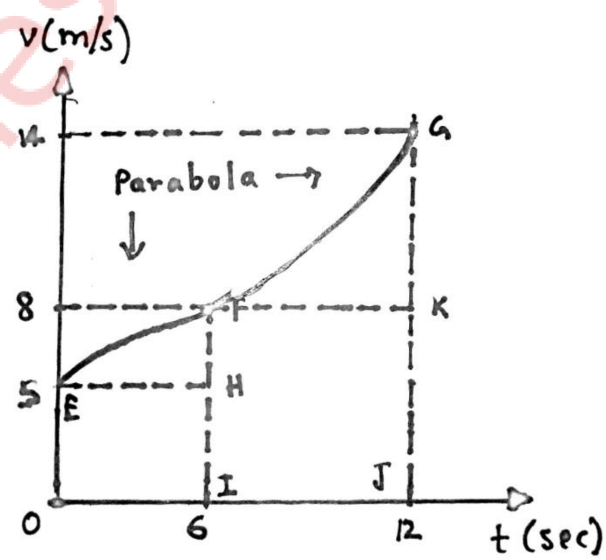
BC on a-t graph represents linearly varying acceleration

$v_2 = v_1 + A(\triangle BCD)$

$= 8 + \frac{1}{2} \times (12-6) \times 2$

$= 14$ m/s

The velocity time graph is drawn below :



FOR DISPLACEMENT TIME GRAPH :

Area under v-t graph gives the displacement

Area under EF = A(EFH) + A(□EHIO)

$=\frac{2}{3}$ x 6 x (8-5) + 6x 5

= 42 m

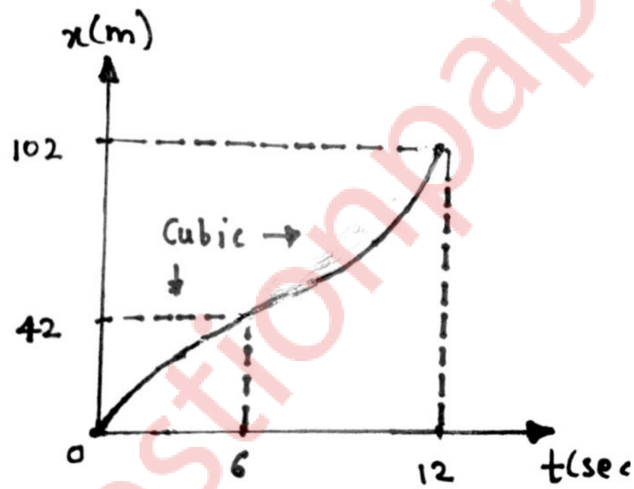
Area under FG = A(GFH) +A(□FIJK)

$=\frac{1}{3}$ x (12 – 6) x (14-8) + (12 – 6) x 8

=12 + 48

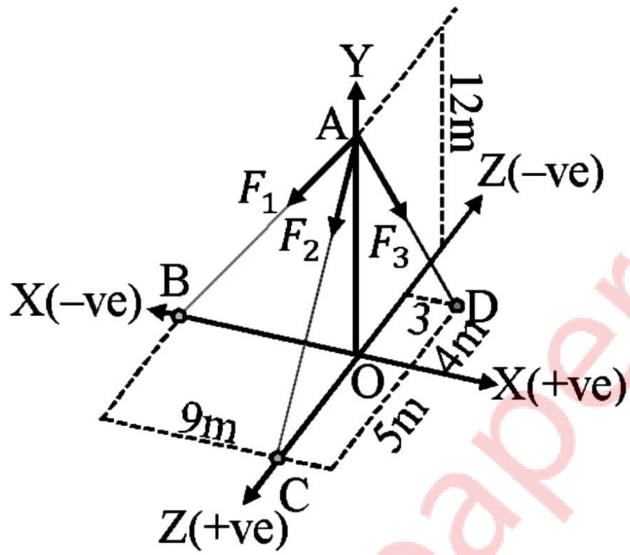
= 60 m

The displacement time graph is shown below :



Q.2(c)The resultant of the three concurrent space forces at A is $\overline{R} = (-788\bar{j})$ N.

Find the magnitude of F1,F2 and F3 forces. (6 marks)



Solution :

Given : A=(0,12,0)

B=(-9,0,0)

C=(0,0,5)

D=(3,0,-4)

Resultant of forces = $(-788\bar{j})$ N

To find : Magnitude of forces F1,F2,F3

Solution:

Assume $\bar{a}$,$\bar{b}$,$\bar{c}$ and $\bar{d}$ be the position vectors of points A,B,C and D respectively w.r.t origin O

$\overline{OA} = \bar{a} = 12\bar{j}$

$\overline{OB} = \bar{b} = -9\bar{i}$

$\overline{OC} = \bar{c} = 5\bar{k}$

$\overline{OD} = \bar{d} = 3\bar{i} - 4\bar{k}$

$\overline{AB}$ = $\bar{b}$ - $\bar{a}$

= -9$\bar{\imath}$-12$\bar{\jmath}$

$\overline{AC}$ = $\bar{c}$ – $\bar{a}$ = 5$\bar{k}$ - 12$\bar{\jmath}$

$\overline{AD}$ = $\bar{d}$ - $\bar{a}$ = 3$\bar{\imath}$ -12$\bar{\jmath}$ - 4$\bar{k}$

Sr.no.	Vector	Magnitude
1.	$\overline{AB}$	15
2.	$\overline{AC}$	13
3.	$\overline{AD}$	13

Sr.no	Vector	Unit vector=$\frac{vector}{Magnitude\ of\ vector}$
1.	$\overline{AB}$	$\frac{-3}{5}\tilde{\imath}$ - $\frac{4}{5}\bar{\jmath}$
2.	$\overline{AC}$	$\frac{-12}{13}\bar{\jmath}$ + $\frac{5}{13}\bar{k}$
3.	$\overline{AD}$	$\frac{3}{13}\bar{\imath}$ - $\frac{12}{13}\bar{\jmath}$ - $\frac{4}{13}\bar{k}$

Force along $\overline{AB}$ = $\overline{F1}$ = F1($\frac{-3}{5}\bar{\imath}$-$\frac{4}{5}\bar{\jmath}$)

Force along $\overline{AC}$ = $\overline{F2}$ = F2($\frac{-12}{13}\bar{\jmath}$ +$\frac{5}{13}\bar{k}$)

Force along $\overline{AB}$ = $\overline{F3}$ = F3($\frac{3}{13}\bar{\imath}$-$\frac{12}{13}\bar{\jmath}$-$\frac{4}{13}\bar{k}$)

Resultant force($\bar{R}$) = $\overline{F1}$+$\overline{F2}$+$\overline{F3}$

-788$\bar{\jmath}$= F1($\frac{-3}{5}\bar{\imath}$-$\frac{4}{5}\bar{\jmath}$)+ $\overline{F2}$($\frac{-12}{13}\bar{\jmath}$ +$\frac{5}{13}\bar{k}$)+ $\overline{F3}$($\frac{3}{13}\bar{\imath}$-$\frac{12}{13}\bar{\jmath}$-$\frac{4}{13}\bar{k}$)

0$\bar{\jmath}$-788$\bar{\jmath}$+0$\bar{k}$= $\overline{F1}$($\frac{-3}{5}\bar{\imath}$-$\frac{4}{5}\bar{\jmath}$)+ $\overline{F2}$($\frac{-12}{13}\bar{\jmath}$ +$\frac{5}{13}\bar{k}$)+ $\overline{F3}$($\frac{3}{13}\bar{\imath}$-$\frac{12}{13}\bar{\jmath}$-$\frac{4}{13}\bar{k}$)

Comparing the equation on both sides

$\frac{-3F1}{5}$+ $\frac{3F3}{13}$=0 ………..(1)

-$\frac{4F1}{5}$--$\frac{12F2}{13}$-$\frac{12F2}{13}$=-788 ………(2)

$\frac{5F2}{13}$ -$\frac{4F3}{13}$ = 0 ……………(3)

Solving (1),(2) and (3)

F1=153.9063 N

F2=320.125 N

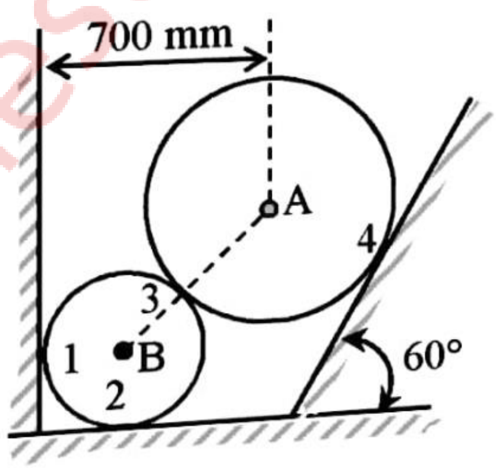
F3=400.1563 N

Answer

Sr.no.	Force	Magnitude
1.	F1	153.9063 N
2.	F2	320.125 N
3.	F3	400.1563 N

Q.3(a)Two spheres A and B of weight 1000N and 750N respectively are kept as shown in the figure..Determine reaction at all contact points 1,2,3 and 4.

Radius of A is 400 mm and radius of B is 300 mm. (8 marks)



Solution :

Given : Two spheres are in equilibrium

W_1=1000 N

W_2=750 N

r_A=400 mm

r_B=300 mm

To find : Reaction forces at contact points 1,2,3 and 4

Solution:

BC = BP = 300mm = 0.3m

AP = 400 mm = 0.4 m

AB = AP + BP

= 0.7m

CO = BC + BO

0.7 = 0.3 + BO

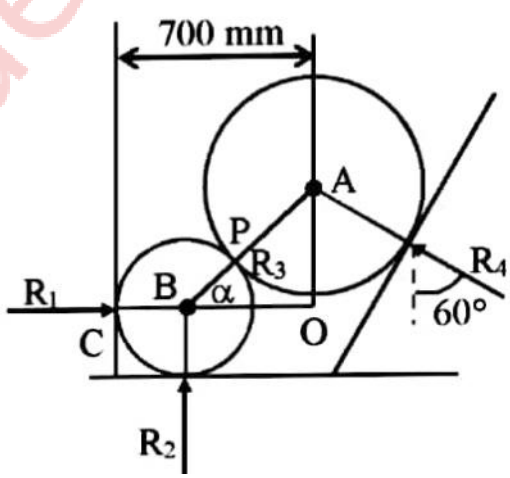
BO = 0.4m

In △AOB

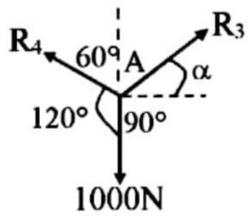
$\cos \boldsymbol{\alpha} = \frac{BO}{AB} = \frac{0.4}{0.7}$

$\alpha = \cos^{-1}(0.5714)$

$\boldsymbol{\alpha = 55.1501^o}$



Forces R_3,R_4 and 1000N are under equilibrium at point A



Applying Lami's theorem

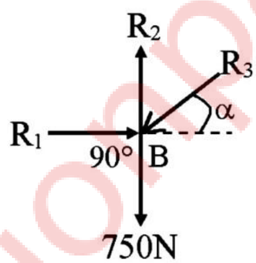
$$\frac{R3}{sin120} = \frac{1000}{\sin(150-\boldsymbol{\alpha})} = \frac{R4}{\sin(90+\boldsymbol{\alpha})}$$

$$\frac{R3}{sin120} = \frac{1000}{\sin(150-55.1501)} = \frac{R4}{\sin(90+55.1501)}$$

Solving the equations

R_3 = 869.1373 N

R_4 = 573.4819 N



Forces R_1,R_2,R_3 and 750N are under equilibrium at B

Applying conditions of equilibrium

$\Sigma F_Y=0$

$-R_3\sin\alpha-750+R2=0$

$R_2=869.1373\sin55.1501+750$

R_2=1463.2591 N (Acting upwards)

Applying conditions of equilibrium

$\Sigma F_X=0$

$R_1-R_3\cos\alpha=0$

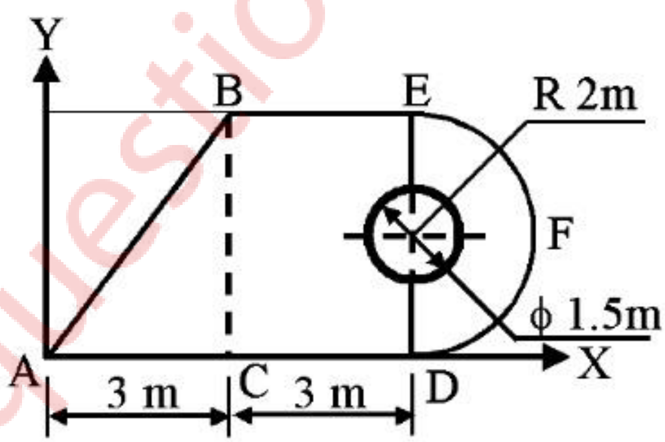
$R_1=869.1373\cos55.1501$

R_1=496.65 N(Acting towards right)

ANSWER :

Sr.no.	**Point**	**Force**
1.	**R_1**	**496.65 N(Towards right)**
2.	**R_2**	**1463.2591 N(Towards up)**
3.	**R_3**	**869.1373 N(55.1501^o in first quadrant)**
4.	**R_4**	**573.4819 N(30^o in second quadrant)**

Q.3(b)A circle of diameter 1.5 m is cut from a composite plate.Determine the centroid of the remaining area of plate. (6 marks)



Solution :

PART	AREA(in m^2)	X co-ordinate of centroid(m)	Y co-ordinate of centroid(m)	$A_x(m^3)$	$A_y(m^3)$
Rectangle	3 x 4 =12	$3 + \frac{3}{2} = 4.5$	2	54	24
Triangle	0.5 x 3 x 4 =6	$3 - \frac{3}{3} = 2$	$\frac{4}{3} = 1.3333$	12	8
Semicircle	$0.5 \times 2^2 \times \pi$ =6.2832	$6+\frac{4 \times 2}{3\pi}$ =6.8488	2	43.0324	12.5664
Circle (To be removed)	$-\pi r2$ $=-0.75^2 \times \pi$ =-1.7671	6	2	-10.6029	-3.5343
Total	22.5161			98.4295	41.0321

X co-ordinate of centroid ($\bar{x}$) = $\frac{\Sigma Ax}{\Sigma A} = \frac{98.4295}{22.5161}$ = 4.3715 m

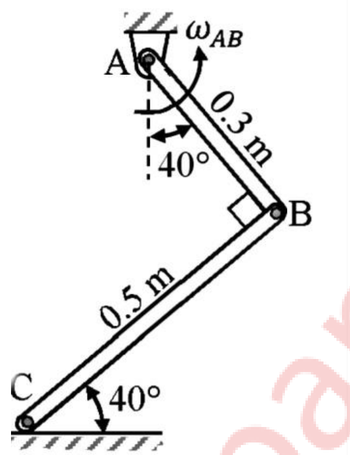
Y co-ordinate of centroid($\bar{y}$) = $\frac{\Sigma Ay}{\Sigma A} = \frac{41.0321}{22.5161}$ = 1.8223 m

Centroid is at (4.3715,1.8223)m

Q.3(c)A rod AB has an angular velocity of 2 rad/sec,counter clock wise as shown.End C of rod BC is free to move on a horizontal surface.Determine:

(1)Angular velocity of rod BC

(2)Velocity of C (6 marks)



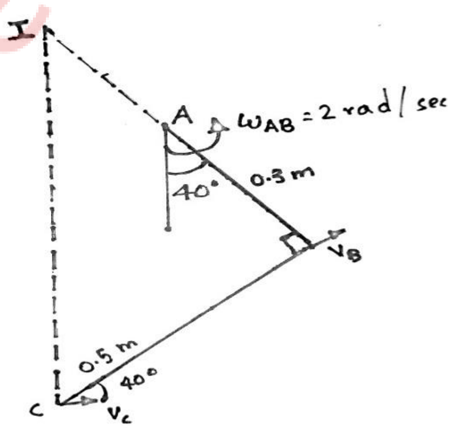
Solution :

Given : ω_{AB}=2 rad/sec (anti clockwise)

To find : ω_{BC}

v_C

Solution :



BY GEOMETRY :

Assume I to be the ICR of rod BC

In △IAB,

∠BIC=40°

∠IBC=90°

$\tan 40 = \frac{BC}{IB} = \frac{0.5}{IB}$

$\sin 40 = \frac{BC}{IC} = \frac{0.5}{IC}$

IB = 0.5959m and IC = 0.7779m

$v_B = r\omega$

$= AB x \omega_{AB}$

$= 0.3 \times 2$

$= 0.6$ m/s

$\omega_{BC} = \frac{vB}{r}$

$= \frac{vB}{IB}$

$= \frac{0.6}{0.5959}$

$=1.0069$ rad/sec

The direction is anti-clockwise

$v_C = r\omega$

$= IC \times \omega_{BC}$

$= 0.7779 \times 1.0069$

$= 0.7832$ m/s

The direction of v_C is towards right

Angular velocity of BC=1.0069 rad/sec(anto clockwise)

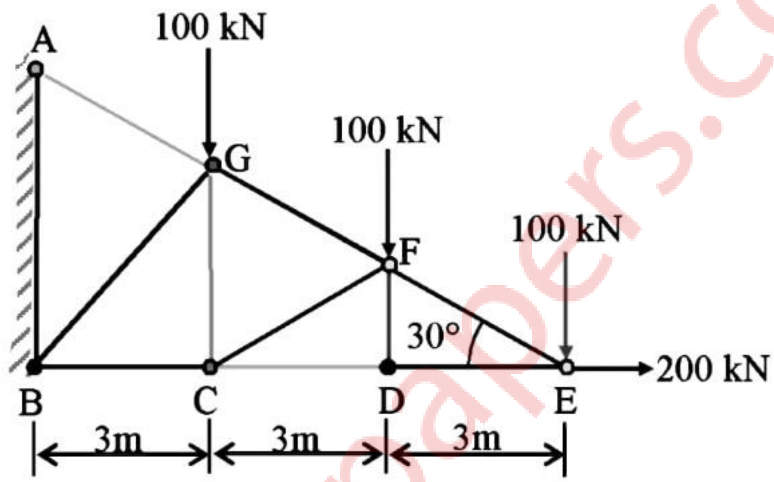
v_C=0.7832 m/s(Towards right)

Q.4(a)A truss is loaded and supported as shown.Determine the following:

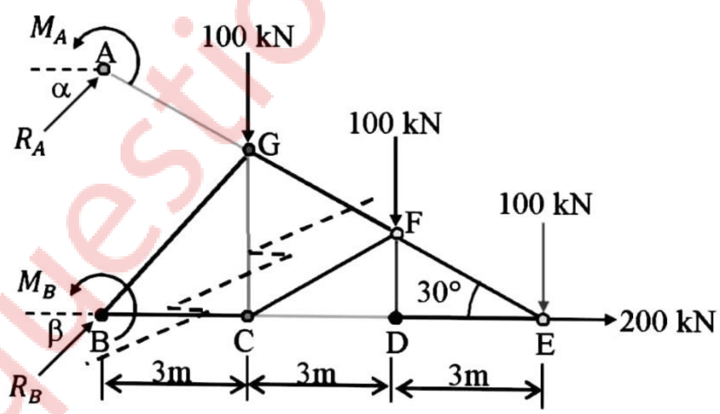
(1)Identify the zero force members,if any

(2)Find the forces in members EF,ED and FC by method of joints.

(3)Find the forces in members GF,GC and BC by method of sections (8 marks)

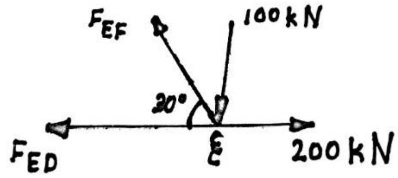


Solution:



By analysis of truss,we can say that **DE is zero force member**

METHOD OF JOINTS:



Joint E:

Applying the conditions of equilibrium

$\Sigma F_Y=0$

$F_{EF}\sin30-100=0$

$F_{EF}=200$ kN

Applying the conditions of equilibrium

$\mathbf{\Sigma F_X=0}$

$-F_{EF}\cos30-F_{ED}+200=0$

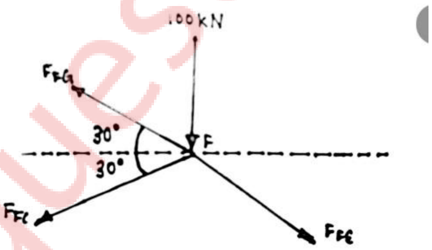
$-200\cos30+200=F_{ED}$

$\mathbf{F_{ED}=26.7949}$ **kN**

△FED is congruent to △FCD

∠FCD=∠FED=30^o

JOINT F:



Applying the conditions of equilibrium

$\mathbf{\Sigma F_Y=0}$

$F_{FG}\sin30-F_{FC}\sin30-F_{FE}\sin30-100=0$

$F_{FG}-F_{FC}-200=200$

F_{FG}-F_{FC}=400 ……….(1)

ΣF_X=0

-F_{FG}cos30-F_{FC}cos30+F_{FE}cos30=0

Dividing by cos30

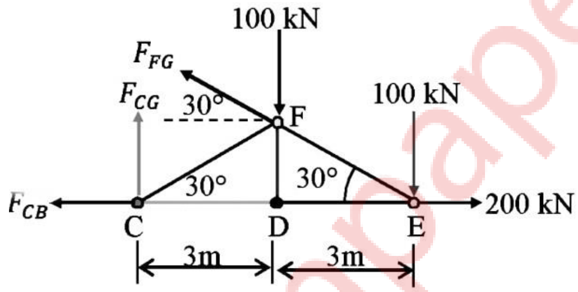
F_{FG} +F_{FC} = 200 ………(2)

Solving (1) and (2)

F_{FG}=300 kN

F_{FC}=-100 kN

METHOD OF SECTIONS:



In △FED

tan 30 = $\frac{FD}{DE}$

DE=3m

FD=$\sqrt{3}$ m

Consider the equilibrium of the truss section

ΣM_C=0

F_{FG}cos 30 x F_D + F_{FG}sin30 x CD -100 x CD -100 X CE = 0

3F_{FG}=900

F_{FG}=300 kN

Applying the conditions of equilibrium

ΣF_X=0

-F_{FG}cos30-F_{CB}+200=0

$-300\cos30+200=F_{CB}$

$\mathbf{F_{CB}=-59.8076\ kN}$

$\Sigma F_Y=0$

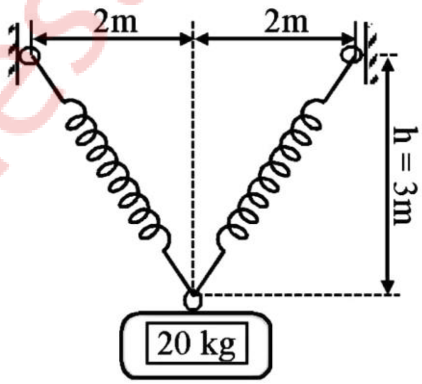
$F_{CG}+F_{FG}\sin30-100-100=0$

$\mathbf{F_{CG}=50\ kN}$

Answer :

Member of truss	Magnitude of force(kN)	Nature of force
BC	59.8076	Compression
GC	50	Tension
GF	300	Tension
FC	100	Compression
ED	26.7949	Tension
EF	200	Tension

Q.4(b)A cylinder has a mass of 20kg and is released from rest when h=0 as shown in the figure.Determine its speed when h=3m.The springs have an unstretched length of 2 m.Take k=40 N/m. (6 marks)



Solution :

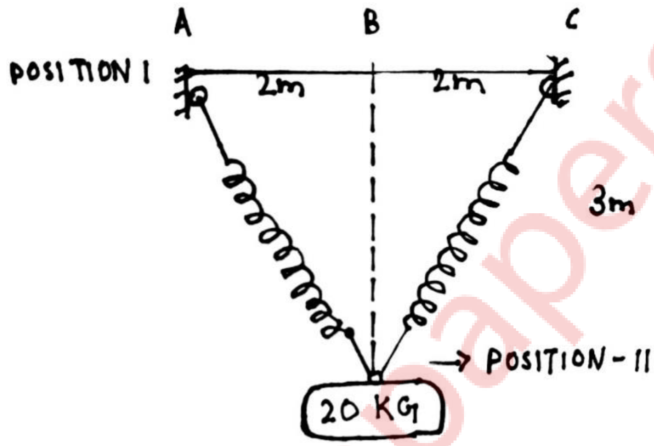
Given : m=20 kg

h=0

k=40 N/m

To find: Speed when h=3m

Solution:



POSITION 1

Un-stretched length of spring = 2 m

Extension (x_1) of spring = 0

Spring energy $=E_{S1} = \frac{1}{2} k x_1^2$

$=0$

PE_1=mgh

=0 J

KE_1=0 J

AT POSITION II:

Let h=-3m

PE_2 = mgh = 20 x 9.81 x (-3)

=-588.6 J

$KE_2 = \frac{1}{2} \times 20v^2$

$=10v^2$

In△ABD,

By Pythagoras theorem

AD=$\sqrt{2^2+3^2}$

=3.6056 m

Extension(x_2) of spring = 3.6056 - 2=1.6056 m

$E_{S2} = \frac{1}{2}kx_2^2 = \frac{1}{2} \times 40 \times 1.6056^2$

=51.5559 J

Applying work-energy principle

$U_{1\text{-}2} = KE_2 - KE_1$

$PE_1 - PE_2 + E_{S1} - E_{S2} = KE_2 - KE_1$

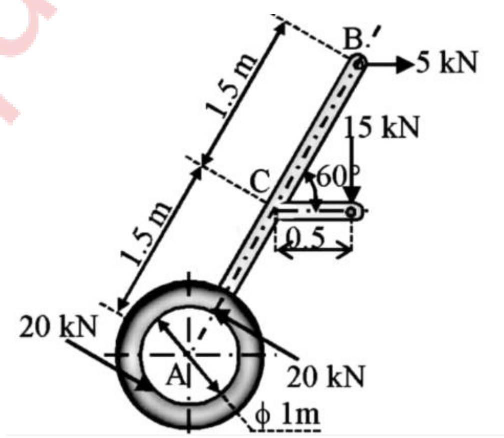
$588.6 - 51.5559 = 10v^2$

v = 7.3283 m/s

Speed when h=3m is 7.3283 m/s

Q.4(c)A machine part is subjected to forces as shown.Find the resultant of forces in magnitude and in direction.

Also locate the point where resultant cuts the centre line of bar AB. (6 marks)



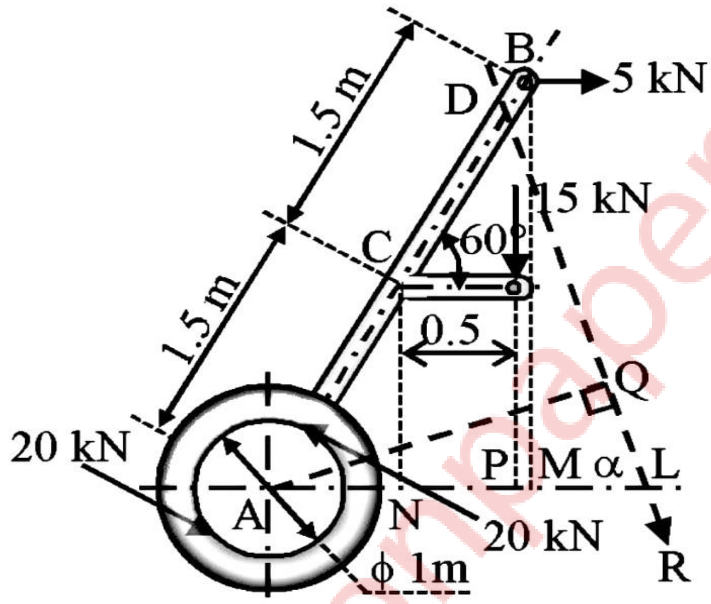
Solution:

Given : A machine subjected to various forces

To find : Resultant of forces

Point where the resultant force cuts the bar AB

Solution:



In △BAM, ∠A=60°

AB=3 m

BM=3sin60

=2.5981 m

In △CAN

AC=1.5m

AN=1.5cos60

=0.75 m

AP=AN+NP

=0.75+0.5

=1.25 m

Two 20N forces are equal and opposite in direction.Hence,they form a couple

Perpendicular distance between two 20 N forces = 1m

Moment of couple = 20 x 1

=20 kN-m (Anti clockwise)

Assume R is the resultant of the forces and it is inclined at an angle θ with horizontal

ΣF_X=5 kN

ΣF_Y=-15 kN

R=$\sqrt{F_x^2 + F_Y^2}$

=$\sqrt{5^2 + (-15)^2}$

=15.8114 kN

θ = $\tan^{-1}(\frac{R_y}{R_x})$

=$\tan^{-1}(\frac{-15}{5})$

=71.5651^o (in fourth quadrant)

Assume that the resultant cut the center line of bar AB at point D

Applying Varigon's theorem

$\Sigma M_A = \Sigma M_A^R$

-5 x BM -15 x AP +20 = R x AQ

-11.7405 = -15.8114 x AQ

AQ = 0.7425 m

In △AQL, ∠ALQ=θ

∠QAL=90 - θ

∠BAL=60

∠QAD=60 - (90 - θ)

= θ - 30

=71.5651 - 30

$=41.5651^o$

InΔDAQ,cos QAD= $\frac{AQ}{AD}$

AD= $\frac{AQ}{\cos DAQ} = \frac{0.7425}{\cos 41.5651} = 0.9924$ m

Resultant force = 15.8114 kN(at 71.5651^o in fourth quadrant)

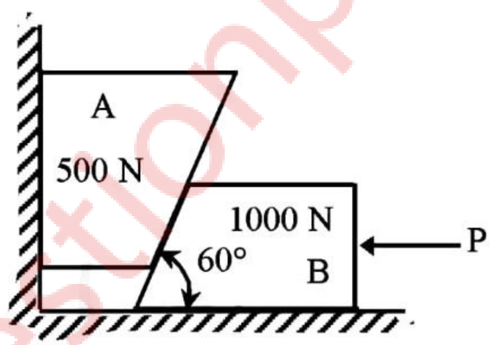
It cuts the center line of bar AB at point D such that AD=0.9924m

---

Q.5(a)Two blocks A and B are resting against the wall and floor as shown in the figure.Find the minimum value of P that will hold the system in equilibrium.

Take μ=0.25 at the floor,μ=0.3 at the wall and μ=0.2 between the blocks.

(8 marks)



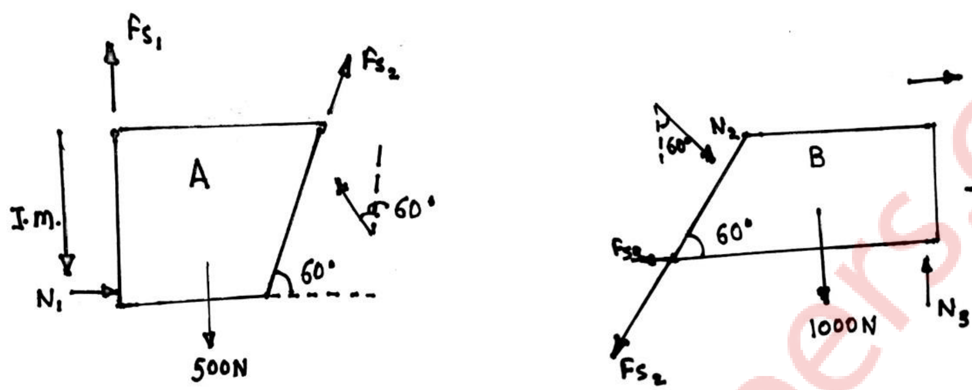
Solution:

Given : μ=0.25 at floor

μ=0.2 between blocks

To find : Minimum value of force P

Solution :



The impending motion of block A is to move down and that of block B is to move towards left

$F_{S1}=\mu 1N_1=0.3N_1$

$F_{S2}=\mu_2N_2=0.2N_2$

$F_{S3}=\mu_3N_3=0.25N_3$ ……………..(1)

Block A is under equilibrium

Applying conditions of equilibrium

$\mathbf{\Sigma F_Y=0}$

$-500+F_{S1}+F_{S2}\sin 60+N_2\cos 60=0$

$0.3N_1+0.6732N_2=500$ ………….(2)

Similarly,

$\mathbf{\Sigma F_X=0}$

$N_1+F_{S2}\cos 60-N_2\sin 60=0$

$N_1+0.2N_2 \times 0.5 - N_2 \times 0.866 = 0$ (From 1)

$N_1-0.766N_2=0$ ……….(3)

Solving (2) and (3)

$\mathbf{N_1=424.1417\ N}$

$\mathbf{N_2=553.71\ N}$

Applying conditions of equilibrium on block B

$\Sigma F_Y=0$

$-1000+N3-F_{S2}\sin60-N_2\cos60=0$

$N_3-0.6732N_2=1000$

$N_3=1372.7576$ N

$\Sigma F_X=0$

$-P-F_{S3}-F_{S2}\cos60+N_2\sin60=0$

$-0.25N_3-0.2N_2 \times 0.5 + N_2 \times 0.866 = P$

P=80.9525 N

The minimum value of force P that will hold the system in equilibrium is 80.9525 N

Q.5(b)A shot is fired with a bullet with an initial velocity of 20 m/s from a point 10 m infront of a vertical wall 5 m high.

Find the angle of projection with the horizontal to enable the shot to just clear the wall.

Also find the range of the shot where the bullet falls on the ground. (6 marks)

Solution :

Given : u=20 m/s

Distance from wall=10m

Height of wall=5m

To find :Angle of projection

Range of shot

Solution:

Let α be the angle of projection of projectile

Equation of projectile is given by:

y=xtanα - $\frac{gx^2}{2u^2}$sec^{2}α

(10,5) are the co-ordinates of top of wall when O is taken as origin

Substituting x=10 and y=5 in the projectile equation

5=10tanα-$\frac{g10^2}{2\ x\ 20^2}$sec^{2}α

1.2262tan2 α-10tan α+6.2262=0

Solving the quadratic equation

tan α=7.4758 or tan α=0.6792

α=82.381^o or α=34.184^o

Range of a projectile is given by R = $\frac{u^2 sin2\alpha}{g}$

Substituting α=82.381^o or α=34.184^o

R=$\frac{20^2\ \sin(2x82.381)}{9.81}$

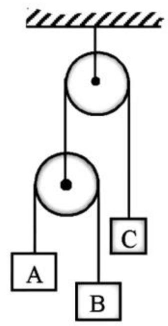
=10.7161 m

R=$\frac{20^2\ \sin(2\ x\ 34.184)o\backslash}{9.81}$

=37.902 m

Angle of projectile should be 82.381^o or 34.184^o and the corresponding ranges will be 10.7161 m and 37.902 m respectively.

Q.5(c)Three blocks A,B and C of masses 3 kg,2 kg and 7 kg respectively are connected as shown.Determine the acceleration of A,B and C.Also find the tension in the string. (6 marks)



Solution :

Given : m_A=3kg

m_B=2 kg

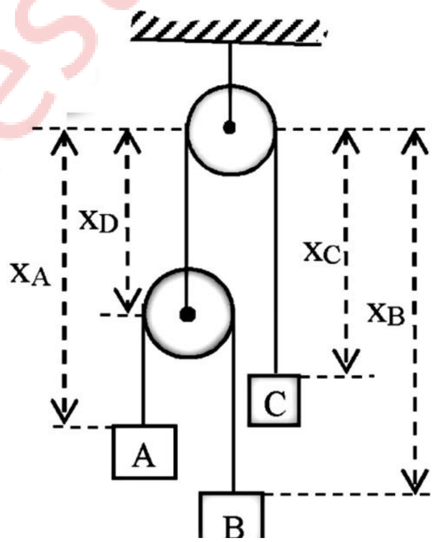
m_C=7kg

To find: Acceleration of blocks A,B and C

Solution:

Assuming the pulleys and the connecting inextensible strings are massless and frictionless

Assume x_A,x_B,x_C and x_D be the displacements of blocks A,B,C and D respectively.



Assume blocks A,B,C and D move downwards.So x_A,x_B,x_C and x_D will increase

Assume k be the length of string that remains constant irrespective of positions of A,B and C.

As the length of string is constant

$(x_A-x_D)+(x_B-x_D)+k=0$

$x_A+x_B-2x_D+k=0$

Differentiating w.r.t t

$v_A+v_B-2v_D=0$

Differentiating once again w.r.t to t

$a_A+a_B-2a_D=0$ …….(1)

and $x_D+x_C+k=0$

$x_D=-x_C-k$

Differentiating w.r.t t

$v_D=-v_C$

Differentiating once again w.r.t to t

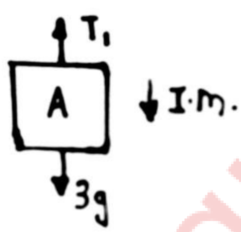
$a_D=-a_C$ ………..(2)

Substituting (2) in (1)

$a_A+a_B+2a_C=0$ …..(3)

Assume tensions T_1 and T_2 be the tensions in two strings

For block A

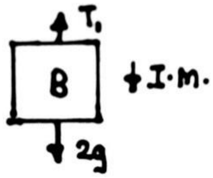


$\Sigma F=m_A a_A$

$3g-T_1= m_A a_A$

$T_1=3g-3a_A$ ……….(4)

For block B



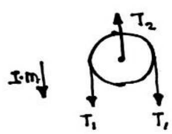
$\Sigma F = m_B a_B$

$2g - T_1 = m_B a_B$

$2g - (3g - 3a_A) = 2a_B$ (From 4)

$3a_A - 2a_B = g$ ……….(5)

For pulley D



$\Sigma F = m_B a_B$

$2T_1 - T_2 = m_D a_D$

$m_D = 0$

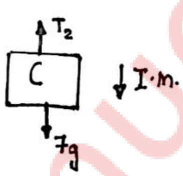
$2T_1 - T_2 = 0$

$T_2 = 2T_1$

$= 2(3g - 3a_A)$

$= 6g - 6a_A$ ………(6) (From 6)

For block C



$\Sigma F = m_B a_B$

$7g - T_2 = m_C a_C$

$7g - (6g - 6a_A) = 7a_C$ ………..(From 6)

$6a_A - 7a_C = -g$ ....(7)

Solving (3),(5) and (7)

$a_A = 0.4988\ m/s^2$

$a_B = -4.1568\ m/s^2$

$a_C = 1.8290\ m/s^2$

From (4)

$T_1 = 3g - 3a_A$

$= 3(9.81 - 0.4988)$

$= 27.9336\ N$

From (6)

$T_2 = 2T_1$

$= 55.8671\ N$

Acceleration of block A=0.4988 m/s^2(Vertically downwards)

Acceleration of block B=4.1568 m/s^2(Vertically upwards)

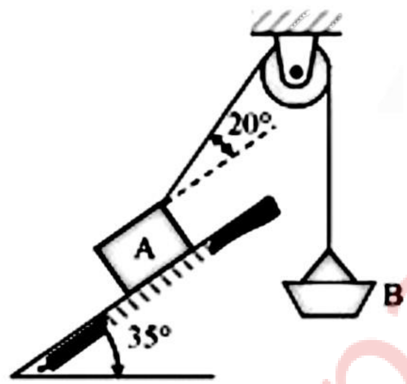
Acceleration of block C=1.8290 m/s^2(Vertically downwards)

Tension of the string T_1=27.9336 N

Q.6(a)Block A of weight 2000N is kept on the inclined plane at 35°.It is connected to weight B by an inextensible string passing over smooth pulley.

Determine the weight of pan B so that B just moves down.Assume μ=0.2.

(5 marks)



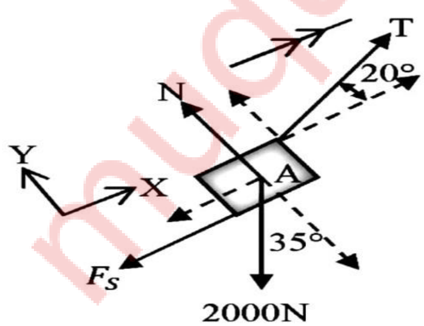
Given : Weight of block A=2000N

Angle of inclined plane = 35°

μ=0.2

To find : Weight of pan B

Solution :



The pan B is in equilibrium

Applying the conditions of equilibrium

$\Sigma F_Y=0$

$T-W_B=0$

$T=W_B$ ………..(1)

Applying the conditions of equilibrium on block A

$\Sigma F_Y=0$

$N-W_A\cos35+T\sin20=0$

From (1)

N=2000cos35-WBsin20 ………..(2)

$F_S=\mu_s N$

$F_S=0.2(2000\cos35-WB\sin20)$

$F_S=400\cos35-0.2WB\sin20$

Applying the conditions of equilibrium on block A

$\Sigma F_X=0$

$T\cos20-W_A\sin35-F_S=0$

$W_B\cos20-2000\sin35-(400\cos35-0.2W_B\sin20)=0$(From 1 and 2)

$W_B=\frac{2000sin35+400cos35}{cos20+0.2sin20}$

W_B=1462.9685 N

The weight of pan B so that pan B just moves down is 1462.9685 N

Q.6(b)A particle falling under gravity travels 25 m in a particular second. Find the distance travelled by it in the next 3 seconds. (4 marks)

Solution :

Given : Particle falls 25 m in a particular second

$S_n = -25$ m

$u = 0$ m/s

To find : Distance travelled by it in next 3 seconds

Solution:

Distance travelled by the particle in nth second is

$S_n = u + \frac{1}{2}\, a\,(2n - 1)$

$-25 = 0 - \frac{1}{2} \times 9.81 \times (2n-1)$

$5.0968 = 2n-1$

n = 3.0484

Considering n as an integer

n = 3 s

Using kinematical equation : $s = ut + \frac{1}{2}at^2$ ……………..(1)

S is the displacement of the particle in 3 seconds

$S = 0 - \frac{1}{2} \times 9.81 \times 3^2$

$S = -44.145$ m

V is the displacement of particle in 6 seconds is

$V = 0 - \frac{1}{2} \times 9.81 \times 6^2$ (From 1)

$= -176.58$ m

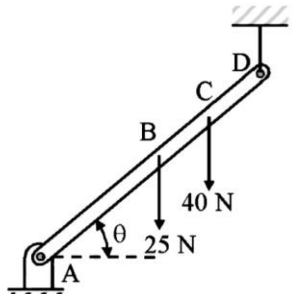
The distance travelled by particle in 4^{th}, 5^{th} and 6^{th} seconds = 176.58-44.145

=132.435 m

The distance travelled by particle in next 3 seconds is 132.435 m

Q.6(c)A rod AD of length 40 cm is suspended from point D as shown in figure.

If it has a weight of 25 N and also supports a load of 40N,find the tension in the cable using the method of virtual work.Take AC=30 cm.



Solution:

Given : Length of rod AD=40cm=0.4m

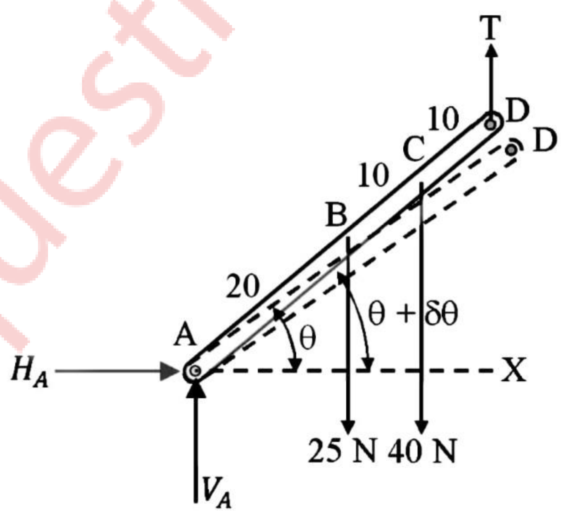
AC=0.3m

W=25N

Load on rod AD=40N

To find : Tension in the cable

Solution:



Assume rod AD have a small virtual angular displacement $\delta\theta$ in the clockwise direction

T is the tension in the cable

Assume A be the origin and AX be the X-axis

Reaction forces HA and VA do not do any virtual work

Sr. no.	Active force	Co-ordinate of the point of action along the force	Virtual displacement
1.	W = 25N	0.2sinθ	δy_B=0.2cosθ δθ
2.	40 N	0.3sinθ	δy_C=0.3cosθ δθ
3.	T	0.4sinθ	δy_D=0.4cos θδθ

By using the principle of virtual work,

$\delta U=0$

$-25 \times \delta y_B - 40 \times \delta y_C + T \times \delta y_D = 0$

$T \times \delta y_D = 25 \times \delta y_B + 40 \times \delta y_C$

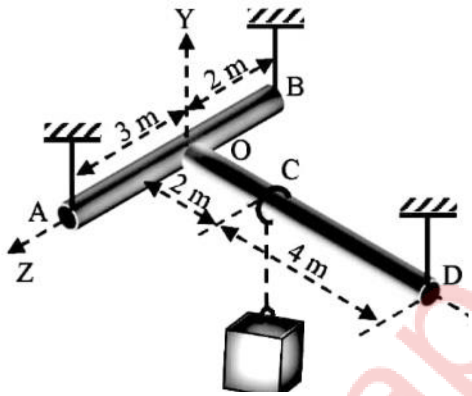
$T \times (0.4\cos\theta\delta\theta) = 25 \times (0.2\cos\theta\,\delta\theta) + 40 \times (0.3\cos\theta\,\delta\theta)$

Dividing by cosθ δθ and solving

T=42.5N

Tension in the cable=42.5N

Q.6(d)A T-shaped rod is suspended using 3 cables as shown.Neglecting the weight of rods,find the tension in each cable.



Solution:

Given: A T shaped suspended with cables supporting a bock of 100 N is in equilibrium

To find: Tension in the cables

Solution:

Applying the conditions of equilibrium

$\Sigma F_y = 0$

T1+T2+T3-100=0

T1+T2+T3=100 …………(1)

Consider moment about an axis which is parallel to X axis and it is passing through point A

ΣMx=0

T2 X AB – 100 XAO +T3 X AO = 0

5T2+3T3=300 ………..(2)

Consider moment about Z axis at point O

ΣMZ=0

-100 X CO+T3XDO=0

6T3=200

T3=33.3333 N ………..(3)

From (2) and (3)

5T2 + 3 x 33.3333 =300

5T2=200 N

T2=40 N ……(4)

From (1),(3) and (4)

T1+40+33.3333=100

T1=26.6667 N

T1=26.6667 N

T2=40 N

T3=33.3333 N

MUMBAI UNIVERSITY

SEMESTER-1

ENGINEERING MECHANICS SOLVED PAPER-MAY 2017

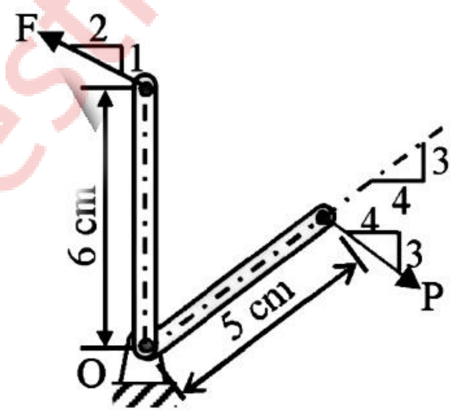
N.B:-(1)Question no.1 is compulsory.

(2)Attempt any 3 questions from remaining five questions.

(3)Assume suitable data if necessary,and mention the same clearly.

(4)Take g=9.81 m/s^2,unless otherwise specified.

Q.1(a) In the rocket arm shown in the figure the moment of 'F' about 'O' balances that P=250 N.Find F. (4 marks)

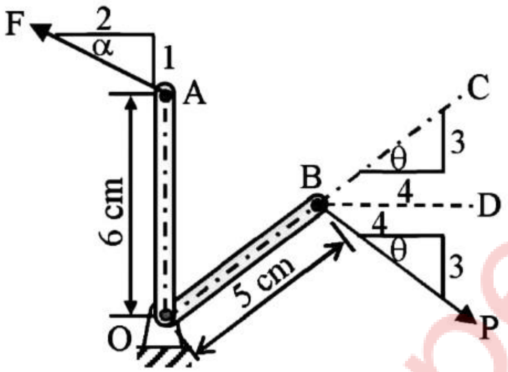


Solution :

Given : P = 250 N

To find : Magnitude of force F

Solution :



tan **α** = $\frac{1}{2}$

= 0.5

α = 26.5651°

tan θ = $\frac{DE}{AD} = \frac{DE}{BC} = \frac{3}{4}$ = 0.75

θ = 36.87°

∠CBD = ∠PBD = θ = 36.87°

∠CBP=2 θ = 2 x 36.87 = 73.74°

It is given that at O the moment of F about O balances the moment of P

Fcos α x OA = Psin2 θ x OB

Fcos26.5651 x 6 = 250sin 73.74 x 5

F=223.6068 N

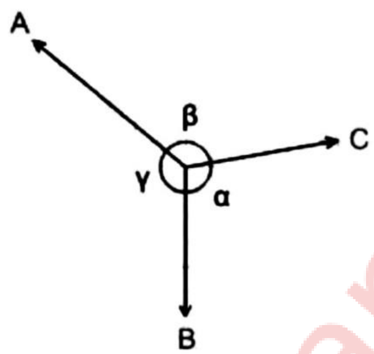
Magnitude of force F= 223.6068 N

Q.1(b) State Lami's theorem.

State the necessary condition for application of Lami's theorem. (4 marks)

Answer :

Lami's theorem states that if three forces acting at a point are in equilibrium, each force is proportional to the sine of the angle between the other two forces.



According to Lami's theorem, the particle shall be in equilibrium if :

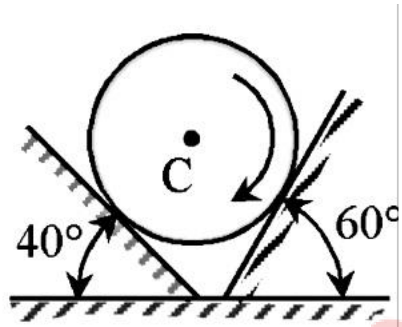
$$\frac{A}{\sin \alpha} = \frac{B}{\sin \beta} = \frac{C}{\sin \gamma}$$

The conditions of Lami's theorem are:

(1)Exact 3 forces must be acting on the body.

(2)All the forces should be either converging or diverging from the body.

Q.1(c)A homogeneous cylinder 3 m diameter and weighing 400 N is resting on two rough inclined surface's shown.If the angle of friction is 15°.Find couple C applied to the cylinder that will start it rotating clockwise. (4 marks)



Solution :

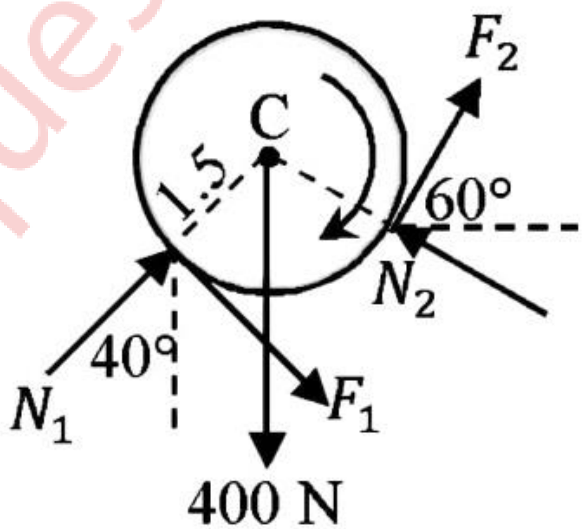
Given : Angle of friction is 15o

μ = tan 15 = 0.2679

Radius = 1.5 m

To find : Couple C

Solution:



$F_1=\mu N_1=0.2679N_1$ ………………….(1)

$F_2=\mu N_2=0.2679N_2$ ………….……….(2)

Assuming the body is in equilibrium

ΣFx=0

$F_1\cos40+N_1\sin40+F_2\cos60-N_2\sin60=0$

$N_1(0.2679\cos40 + \sin40)+N_2(0.2679\cos60-\sin60)=0$ …………(3)

ΣFy=0

$-F_1\sin40+N_1\cos40+F_2\sin60+N_2\cos60-400=0$

$N_1(-0.2679\sin40+\cos40)+N_2(0.2679\sin60+\cos60)=400$ ………..(4)

Solving (3) and (4)

N_1=277.4197 N and N_2=321.3785 N

Substituting N1 and N2 in (1 and 2)

$F_1=0.2679 \times 277.4197 = 74.3344$ N

$F_2=0.2679 \times 321.3785 = 86.1131$ N …….(5)

C is the couple required to rotate the cylinder clockwise

$C=F_1 \times r + F_2 \times r$

= 240.6712 Nm(clockwise) (r=1.5 m)(From 5)

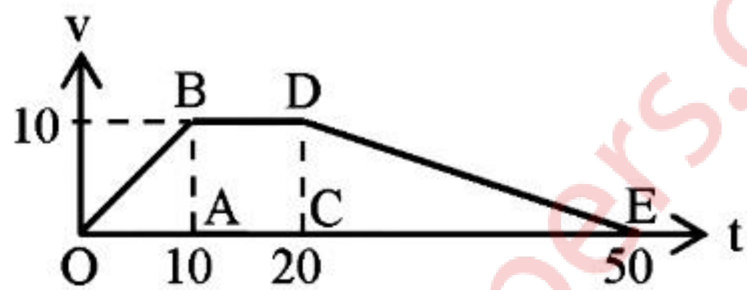
The couple C required to rotate the cylinder clockwise is 240.6712 Nm(clockwise)

Q.1(d) From (v-t) diagram find

(1) Distance travelled in 10 second.

(2) Total distance travelled in 50 second.

(3) Retardation (4 marks)



Solution:

We know that the area under v-t graph gives the distance travelled

DISTANCE TRAVELLED IN 0 TO 10 sec = A(△OAB)

$= \frac{1}{2}$ x OA x AB

$= \frac{1}{2}$ x 10 x 10

= 50 m

DISTANCE TRAVELLED IN 0 TO 50 sec = A(Trapezium OBDE)

$= \frac{1}{2}$ x (OE+BD) x AB

$= \frac{1}{2}$ x (50+10) x 10

= 300 m

CONSIDER THE MOTION FROM 20 sec TO 50 sec

We know that slope of v-t graph gives acceleration

E=(50,0) and D=(20,10)

Slope of line DE= $\frac{0-10}{50-20}$ = $\frac{-1}{3}$ = -0.3333 m/s^2

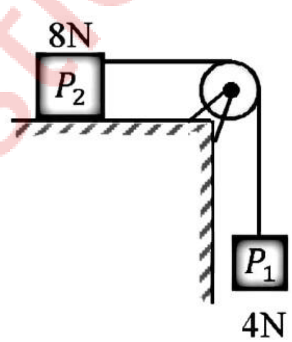
Distance travelled by object in 10 sec = 50 m

Distance travelled by object in 50 sec = 300 m

Acceleration = - 0.3333 m/s2

Q1(e))Blocks P_1 and P_2 are connected by inextensible string.Find velocity of block P_1,if it falls by 0.6 m starting from rest.

The co-efficient of friction is 0.2.The pulley is frictionless. **(4 marks)**



Solution:

Given : P_1 falls by 0.6 m starting from rest

$\mu = 0.2$

To find : Velocity of block P_1

Solution :

Consider the motion of block P_2

Weight of motion $P_2 = 8$ N

Mass of $P_2 = \frac{8}{g}$

P2 has no vertical motion

$\mathbf{\Sigma F_y = 0}$

$N_2 - 8 = 0$

$N_2 = 8$ N

$F_2 = \mu N_2$

$= 1.6$ N

Consider the horizontal motion

$\mathbf{\Sigma F_x = m_2 a}$

$T - F_2 = m_2 a$

$T = 1.6 + \frac{8}{g}a$ ……………(1)

For block P_1
Weight of $P_1 = 4$ N

Mass of $P_1 = \frac{4}{g}$ ……….(2)

For downward motion

$\mathbf{\Sigma F_y = m_1 a}$

$4 - T = m_1 a$

$4 - 1.6 - \frac{8}{g}a = \frac{4}{g}a$ (From 1 and 2)

$a = 1.962$ m/s^2

$\mathbf{v^2 = u^2 + 2as}$

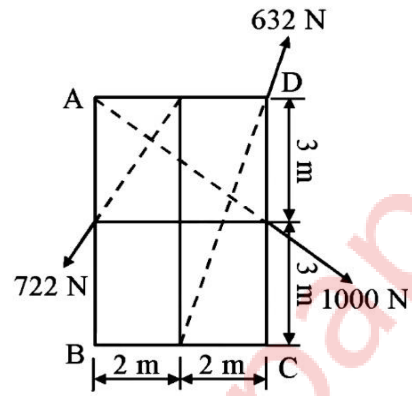
$u = 0$ and $s = 1.6$ m

Substituting the values in equation

v = 1.5344 m/s

Velocity of block P_1=1.5344 m/s (towards down)

Q2(a) Compute the resultant of three forces acting on the plate shown in the figure. Locate it's intersection with AB and BC. (6 marks)

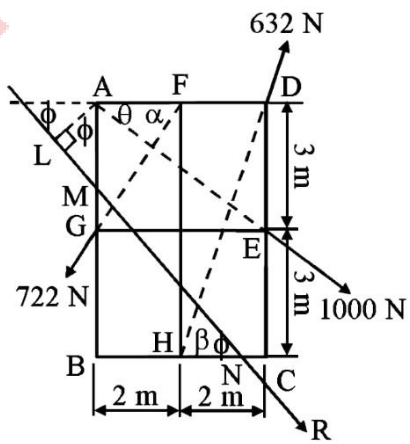


Solution :

Given : Various forces acting on a body

To find : Resultant of the forces and intersection of resultant with AB and BC

Solution :



In △ AFG ,

$\tan\alpha = \frac{AG}{AF} = \frac{DE}{BH} = \frac{3}{2} = 1.5$

$\alpha = \tan^{-1}(1.5) = 56.31^o$

In △DAE,

$\tan\theta = \frac{DE}{AD} = \frac{DE}{BC} = \frac{3}{4} = 0.75$

$\theta = \tan^{-1}0.75 = 36.87^o$

In △DHC

$\tan\beta = \frac{DC}{HC} = \frac{6}{2} = 3$

$\beta = \tan^{-1}(3)$

$\beta = 71.565^o$

Assume R be the resultant of the forces

$\Sigma F_x = -722\cos\alpha + 1000\cos\theta + 632\cos\beta$

= 599.3624 N

$\Sigma F_y = -722\sin\boldsymbol{\alpha} - 1000\sin\theta + 632\sin\beta$

= -601.1725 N

$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2}$

$R = \sqrt{(599.3624)^2 + (-601.1725)^2}$

R=848.9073 N

$\phi = \tan^{-1}(\frac{\Sigma Fy}{\Sigma Fx})$

$= \tan^{-1}(\frac{-601.1725}{599.3624})$

$= 45.0863^o$ (in fourth quadrant)

Let R cut AB and BC at points M and N respectively

Draw AL ⊥ R

Taking moments about point A

M_A = 632 sin β x AD -722cos α x AG

= 632 x sin71.565o x 4 – 722cos56.31° x 3

=1196.7908 Nm

Applying Varigon's theorem

M_A = R x AL

1196.7908 = 848.9073 x AL

AL=1.4098 m

In △AML,

cos Φ = $\frac{AL}{AM}$

cos 45.0863 = $\frac{1.4098}{AM}$

AM = 1.9967 m

MB = AB - AM

= 6 - 1.9967

= 4.0033 m

In △BMN

tan Φ = $\frac{BM}{BN}$

tan 45.0863 = $\frac{4.0033}{BN}$

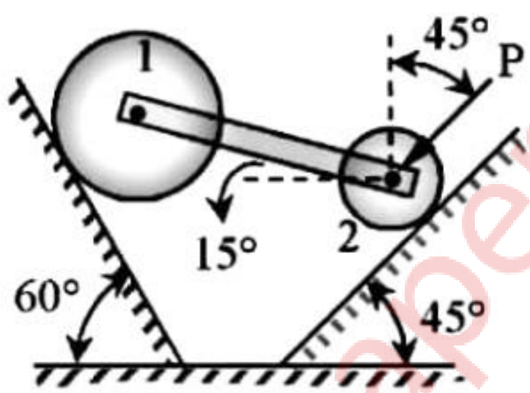
BN = 3.9912 m

R=848.9073 N (45.0863° in fourth quadrant)

Resultant force intersects AB and BC at M and N such that AM=1.9967 m and BN=3.9912 m

Q.2(b) Two cylinders 1 and 2 are connected by a rigid bar of negligible weight hinged to each cylinder and left to rest in equilibrium in the position shown under the application of force P applied at the center of cylinder 2.

Determine the magnitude of force P.If the weights of the cylinders 1 and 2 are 100N and 50 N respectively. (8 marks)



Solution :

Given : $W_1 = 100$ N

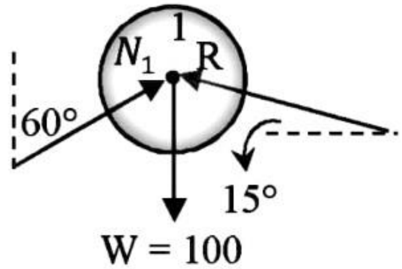
$W_2 = 50$ N

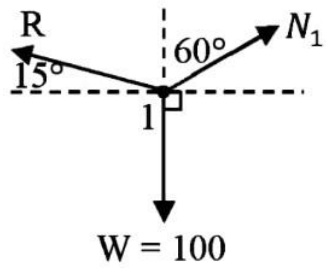
Cylinders are connected by a rigid bar

To find : Magnitude of force P

Solution :

Consider cylinder I





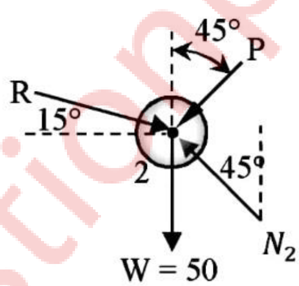
Applying Lami's theorem :

$$\frac{R}{\sin(90+30)} = \frac{W}{\sin(60+75)} = \frac{N_1}{\sin(90+15)}$$

R = $\frac{100}{\sin 135}$ x sin 120

R = 122.4745 N

Cylinder 2 is under equilibrium



Applying conditions of equilibrium

ΣFy = 0

N_2sin45 - Rsin15 - Psin45 – W = 0

N_2sin45 - Psin45 = 122.4745 x 0.2588 + 50

N_2sin45 - Psin45 = 81.6987 ………..(1)

Applying conditions of equilibrium

$\Sigma Fx = 0$

$-N_2\cos45+R\cos15-P\cos45=0$

$N_2\cos45+P\cos 45=118.3013$ ……(2)

Solving (1) and (2)

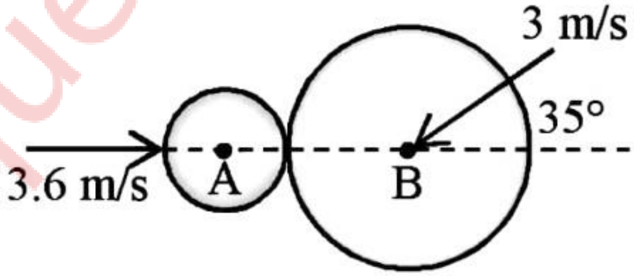
P=25.8819 N

Magnitude of force P required = 25.8819 N

Q.2(c) Just before they collide,two disk on a horizontal surface have velocities shown In figure.

Knowing that 90 N disk A rebounds to the left with a velocity of 1.8 m/s.Determine the rebound velocity of the 135 N disk B.Assume the impact is perfectly elastic.

(6 marks)



Solution :

Given : W_A = 90N

W_B = 135 N

Taking velocity direction towards right as positive and towards left as negative

Initial velocity of disk A= 3.6 m/s

Final velocity of disk A=-1.8 m/s

Initial velocity of disk B= 3 m/s

To find : Rebound velocity of disk B

Solution :

$m_A = \frac{90}{g}$ kg

$m_B = \frac{135}{g}$ kg

Consider the X and Y components of u_B

$u_{BX} = -u_B\cos35 = -2.4575$ m/s

$u_{BY} = -u_B\sin35 = -1.7207$ m/s

APPLYING LAW OF CONSERVATION OF MOMENTUM :

$m_Au_A + m_Bu_B = m_Av_A + m_Bv_B$

$\frac{90}{g}$ x 3.6 + $\frac{135}{g}$ x (-2.4575) = $\frac{90}{g}$ x (-1.8) + $\frac{135}{g}$ x v_{BX}

v_{BX} = 1.1425 m/s

As the impact takes place along X-axis,the velocities of two disks remains same along Y-axis

$v_{BY} = u_{BY} = -1.7207$ m/s

$v = \sqrt{(v_{BX})^2 + (v_{BY})^2}$

$v = \sqrt{1.1425^2 + (-1.7207)^2}$

v = 2.0655 m/s

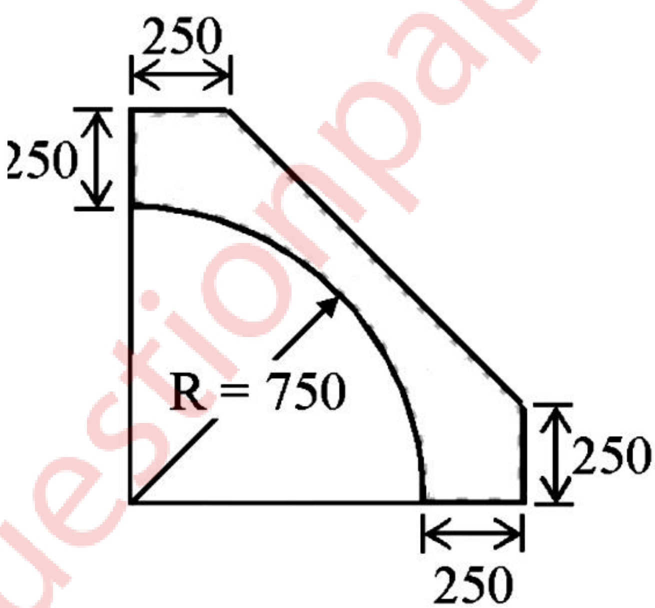
α = $\tan^{-1}(\frac{-1.7207}{1.1425})$

α = 56.4169°

VELOCITY OF DISK B AFTER IMPACT = 2.0655 m/s (56.4169o in fourth quadrant)

Q.3(a) Find the centroid of the shaded portion of the plate shown in the figure.

(8 marks)



Solution :

Y = X is the axis of symmetry

The centroid would lie on this line

Sr.no.	PART	AREA(in mm2)	X co-ordinate(mm)	Ax(mm3)

1.	RECTANGLE	=1000 X 1000 =1000000	$\frac{1000}{2} = 500$	500000000
2.	TRIANGLE (to be removed)	$\frac{1}{2}$ X 750 X 750 = -281250	$1000 - \frac{750}{3}$ = 750	-210937500
3.	QUARTER CIRCLE (To be removed)	$\frac{\pi r^2}{4}$ = 441786.4669	$\frac{4 X 750}{3\pi}$ = 3141.5926	-140625000
	TOTAL	276963.4669		148437500

$\bar{X} = \frac{\Sigma Ax}{\Sigma A} = \frac{148437500}{276963.5331}$ = **535.946 mm**

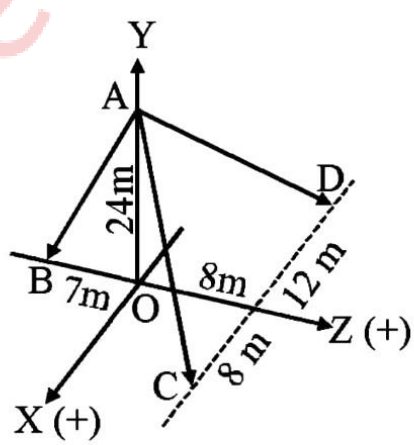
$\bar{y} = \bar{X}$ = **535.946 mm**

CENTROID IS AT (535.946,535.946)mm

Q.3(b) Co-ordinate distance are in m units for the space frame in figure.

There are 3 members AB,AC and AD.There is a force W=10 kN acting at A in a vertically upward direction.

Determine the tension in AB,AC and AD. (6 marks)



Solution :

Given : A = (0,24,0)

B = (0,0,-7)

C = (8,0,8)

D = (-12,0,8)

To find : Tension in AB,AC and AD.

Solution :

Assume $\bar{a},\bar{b},\bar{c},\bar{d}$ be the position vectors of points A,B,C,D with respect to origin O.

$\overline{OA} = \bar{a} = 24\bar{j}$

$\overline{OB} = \bar{b} = -7\bar{k}$

$\overline{OC} = \bar{c} = 8\bar{i} + 8\bar{k}$

$\overline{OD} = \bar{d} = -12\bar{i} + 8\bar{k}$

$\overline{AB} = \bar{b} - \bar{a} = -24\bar{j} - 7\bar{k}$ Magnitude = 25 Unit vector = $\frac{-24j-7k}{25}$

$\overline{AC} = \bar{c} - \bar{a} = 8(\bar{i} - 3\bar{j} + \bar{k})$ Magnitude= $8\sqrt{11}$ Unit vector = $\frac{8(i-3j+k)}{8\sqrt{11}}$

$\overline{AD} = \bar{d} - \bar{a} = 4(-3\bar{i} - 6\bar{j} + 2\bar{k})$ Magnitude = 28 Unit vector = $\frac{4(-3i-6j+2k)}{28}$

Assume T_1,T_2 and T_3 be the tensions along AB,AC and AD

$\mathbf{T_1 = T_1(\frac{-24j-7k}{25})}$

$\mathbf{T_2 = T_2(\frac{8(i-3j+k)}{8\sqrt{11}})}$

$\mathbf{T_3 = T_3(\frac{4(-3i-6j+2k)}{28})}$

A force of 10kN is acting at point A in vertically upward direction

Applying conditions of equilibrium

$\mathbf{10\bar{j} + T_1 + T_2 + T_3 = 0}$

$-10\bar{j} = T_1(\frac{-24j-7k}{25}) + T_2(\frac{8(i-3j+k)}{8\sqrt{11}}) + T_3(\frac{4(-3i-6j+2k)}{28})$

$0\bar{\imath} - 10\bar{j} + 0\bar{k} = T_1(\frac{-24j-7k}{25}) + T_2(\frac{8(i-3j+k)}{8\sqrt{11}}) + T_3(\frac{4(-3i-6j+2k)}{28})$

Comparing both sides of equation

$\frac{T2}{\sqrt{11}} - \frac{3T_3}{7} = 0$

$\frac{-24T_1}{25} - \frac{3T_2}{\sqrt{11}} - \frac{6T_3}{7} = -10$

$\frac{-7T_1}{25} \frac{T_2}{\sqrt{11}} + \frac{2T_3}{7} = 0$

Solving the equations simultaneously

T_1=5.5556 N

T_2=3.0955 N

T_3=2.1778

$T_{AB} = -5.3333\,\bar{j} - 1.5556\bar{k}$

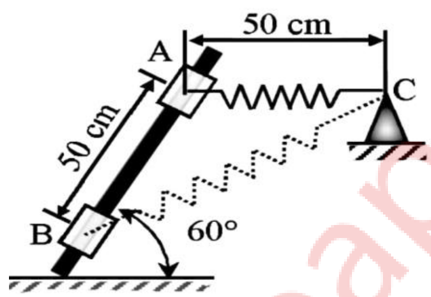
$T_{AC} = 0.9333\bar{\imath} - 2.8\,\bar{j} + 0.9333\bar{k}$

$T_{AD} = -0.9333\bar{\imath} - 1.8667\,\bar{j} + 0.6222\bar{k}$

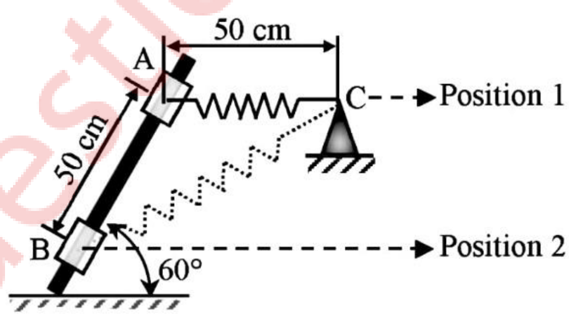
Q.3(c) A 50 N collar slides without friction along a smooth and which is kept inclined at 60^o to the horizontal.

The spring attached to the collar and the support C.The spring is unstretched when the roller is at A(AC is horizontal).

Determine the value of spring constant k given that the collar has a velocity of 2.5 m/s when it has moved 0.5 m along the rod as shown in the figure. (6 marks)



Solution :



Given : W=50 N

AB = AC = 0.5 m

To find : Spring constant

Solution :

Mass of collar = $\frac{50}{g}$ kg

Let us assume that h = 0 at position 2

POSITION 1 :

x = 0

$E_{s1} = \frac{1}{2}$ x k x x_1^2 = 0

h_1 = 0.5sin60 = 0.433 m

PE1=mgh_1=21.65 J

v_A = 0 m/s

KE_1 = 0J

POSITION II :

v_B = 2.5 m/s

PE_2 = mgh = 0 J (because h=0)

$KE_2 = \frac{1}{2}\ X\ m\ v^2 = \frac{1}{2}\ X\ \frac{50}{g}\ X\ 2.5^2$

= 15.9276 J

In △ABC

Applying cosine rule

$BC^2 = AB^2 + AC^2$ -2 X AB X AC X cos(BAC)

$= 0.5^2 + 0.5^2 - 2$ x 0.5 x 0.5 x cos120

= 0.75

BC = 0.866 m

Un-stretched length of the spring = 0.5 m

Extension of spring(x) = 0.866 - 0.5

=0.366 m

$E_{s2} = \frac{1}{2}$ x k x_2^2

= 0.067k

APPLYING WORK ENERGY PRINCIPLE

$U_{1-2} = KE_2 - KE_1$

$PE_1 - PE_2 + E_{S1} - ES_2 = KE_2 - KE_1$

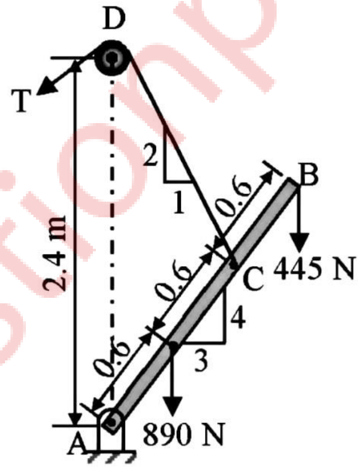
21.6506-0+0-0.067K=15.9276-0

K = 85.4343 N/m

SPRING CONSTANT IS 85.4343 N/m

Q.4(a) A boom AB is supported as shown in the figure by a cable runs from C over a small smooth pulley at D.

Compute the tension T in cable and reaction at A.Neglect the weight of the boom and size of the pulley. (8 marks)



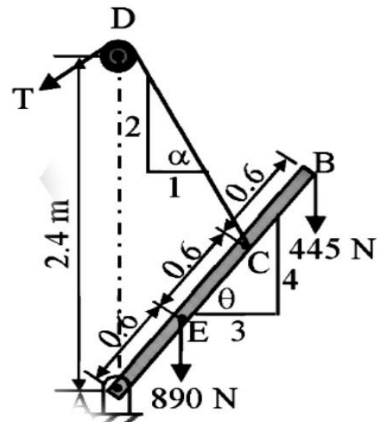
Solution :

Given : Beam AB is supported by a cable

To find : Tension T in cable

Reaction at A

Solution :



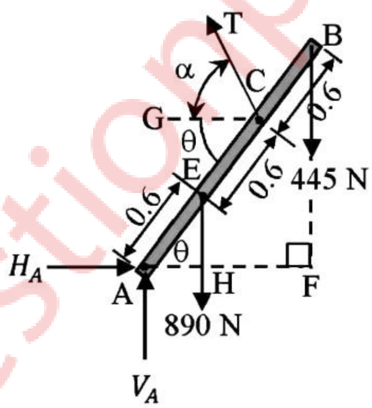
tan α = $\frac{2}{1}$

α = 63.4349°

tan θ = $\frac{4}{3}$

θ = 53.13°

Assume H_A and V_A be the horizontal and vertical reaction forces at A



∠GCA = ∠BAF = θ

∠TCG = α

∠TCA = α + θ

= 63.4349° + 53.16°

= 116.5651°

∠TCB = 180° - 116.5651°

= 63.4349°

AC = AE+EC = 0.6 + 0.6 = 1.2

AB = AC + CB = 1.2 + 0.6 = 1.8

AF = ABcos θ = 1.8cos53.13 = 1.08

AH = AEcos θ = 0.6cos53.13 = 0.36

BEAM AB IS INDER EQUILIBRIUM

Applying conditions of equilibrium

$\Sigma M_A = 0$

-445 X AF – 890 X AH + Tsin63.4349 X AC = 0

T X 0.8944 X 1.2 = 445 X 1.08 + 890 X 0.36

T = 746.2877 N

$\Sigma F_X = 0$

H_A - Tcos63.4349 = 0

H_A=333.75 N

$\Sigma F_Y = 0$

V_A + Tsin63.4349 – 890 – 445 = 0

V_A = 667.5 N

$R_A = \sqrt{H_A^2 + V_A^2}$

$R_A = \sqrt{(333.75)^2 + (667.5)^2}$

R_A = 746.2877 N

$\Phi = \tan^{-1}(\frac{V_A}{H_A})$

$\Phi = \tan^{-1}(\frac{667.5}{333.75})$

Φ =**63.4395°**

Tension in cable = 746.2877 N (63.43949° in second quadrant)

Reaction at A = 746.2877 N (63.4395° in first quadrant)

Q.4(b) The acceleration of the train starting from rest at any instant is given by the expression a=$\frac{8}{v^2+1}$ where v is the velocity of train in m/s.

Find the velocity of the train when its displacement is 20 m and its displacement when velocity is 64.8 kmph. (6 marks)

Solution :

Given : a=$\frac{8}{v^2+1}$

To find : Velocity when displacement is 20 m

Displacement when velocity is 64.8 kmph.

Solution :

a = v$\frac{dv}{dx}$

v$\frac{dv}{dx}$ = $\frac{8}{v^2+1}$

v(v^2+1)dv = 8dx

Integrating both sides

∫ v(v^2+1)dv = ∫ 8dx

$\frac{v^4}{4} + \frac{v^2}{2}$ = 8x + c ……….(1)

Multiplying by 4 on both sides

$V^4 + 2v^2$ = 32x + 4c

Substituting v=0 and x=0 in (1)

c=0

From (1)

$V^4 + 2v^2 = 32x$ ………….(2)

Case 1 : x=20 m

$V^4 + 2v^2 = 32 \text{ x } 20$ ………(From 2)

$V^4 + 2v^2 – 640 = 0$

Solving the equation

$V^2 = 24.3180$

V=4.9361 m/s

Case 2 : V=64.8 kmph(or v = 18 m/s)

$18^4 + 2 \text{ x } 18^2 = 32x$ …………..(From 2)

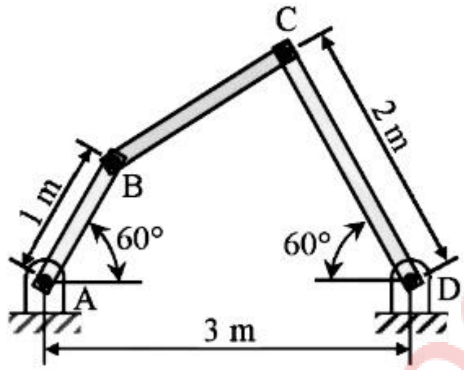
1.5624 = 32x

x = 3300.75 m

When displacement of train is 20 m,then velocity is 4.9361 m/s

When velocity of the train is 64.8 kmph,then its displacement is 3300.75m

Q.4(c) Angular velocity of connector BC is 4 r/s in clockwise direction.What is the angular velocities of cranks AB and CD? (6 marks)



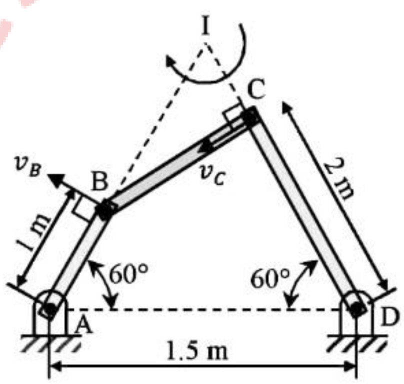
Solution:

Given : Angular velocity of BC is 4 rad/s

To find : Angular velocity of AB and CD

Solution:

ICR is shown in the figure



USING GEOMETRY :

In △IAD

$\angle A = \angle D = 60^o$
$\angle I = 60^o$

$\triangle$ IAD is equilateral

IA = ID = AD = 3 cm

IB + AB = IA

IB = 2 cm

Similarly,we can solve that IC = 1 cm

v = rω

$v_B = IB \times \omega_{BC} = 8$ m/s

$v_C = IC \times \omega_{BC} = 4$ m/s

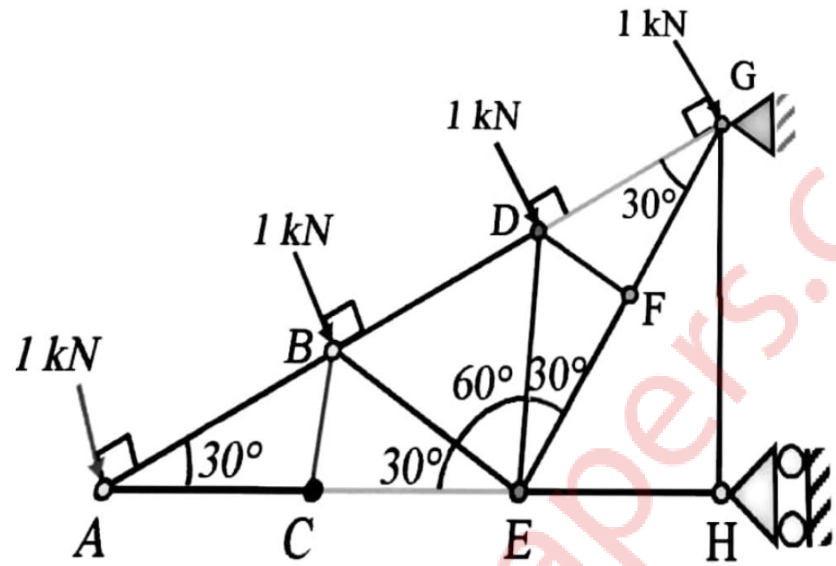
$\omega_{AB} = \frac{v_B}{AB} = \frac{8}{1} = 8$ rad/s(Anti-clockwise)

$\omega_{DC} = \frac{v_c}{DC} = \frac{4}{2} = 2$ rad/s(Anti-clockwise)

Angular velocity of AB=8 rad/s(Anti-clockwise)

Angular velocity of CD=2 rad/s(Anti-clockwise)

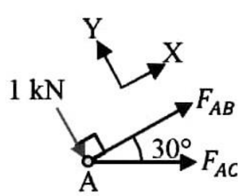
Q.5(a) In the truss shown in figure,compute the forces in each member. (8 marks)



Solution :

We can say that FD,GH and CB are zero force members in the given truss

Joint A :



Applying the conditions of equilibrium

$\Sigma Fy=0$

$-1 – F_{AC} \sin 30 = 0$

F_{AC} = -2kN

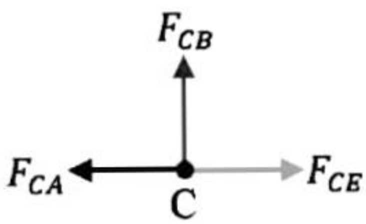
Applying the conditions of equilibrium

$\Sigma Fx = 0$

$F_{AB} + F_{AC} \cos 30 = 0$

F_{AB} = 1.7321 Kn

JOINT C :

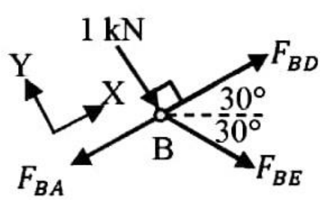


Applying the conditions of equilibrium

$\Sigma Fx = 0$

$F_{CE} = F_{CA}$ = -2kN

JOINT B :



Applying the conditions of equilibrium

$\Sigma Fy = 0$

$-1 - F_{BE} \sin 60 = 0$

FBE = -1.1547 kN

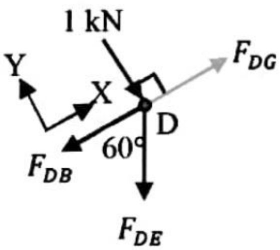
Applying the conditions of equilibrium

$\Sigma Fx = 0$

$-F_{BA} + F_{BE}\cos 60 + F_{BD} = 0$

F_{BD} = 2.3094 kN

JOINT D :



Applying the conditions of equilibrium

$\Sigma Fy = 0$

$-1 - F_{DE}\sin60 = 0$

F_{DE} = -1.1547 kN

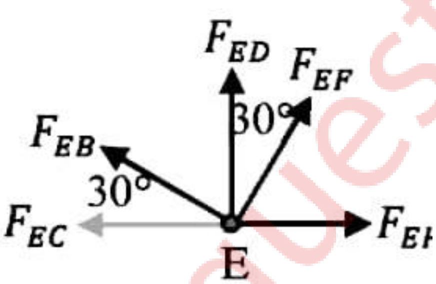
Applying the conditions of equilibrium

$\Sigma Fx = 0$

$-F_{DB} - F_{DE}\cos60 + F_{DG} = 0$

F_{DG} = 1.7321 kN

JOINT E :



Applying the conditions of equilibrium

$\Sigma Fy = 0$

$F_{ED} + F_{EF}\cos30 + F_{EB}\sin30 = 0$

$F_{EF}\cos30 = -(-1.1547)-(-1.1547) \times \frac{1}{2}$

F_{EF} = 2kN

Applying the conditions of equilibrium

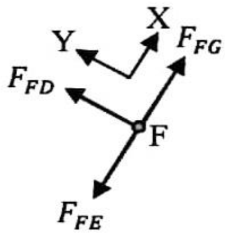
$\Sigma Fx = 0$

$-F_{EC} + F_{EH} + F_{EF}\sin30 - F_{EB}\cos30 = 0$

$F_{EH} = F_{EC} - F_{EF}\sin30 + F_{EB}\cos30$

FEH = -4kN

Joint F :



Applying the conditions of equilibrium

$\Sigma Fx = 0$

$F_{FG} = F_{FE} = -2kN$

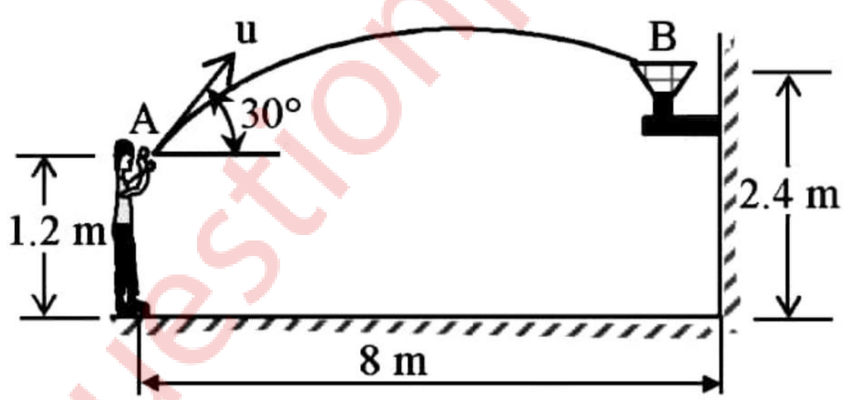
Final answer :

Sr.no.	MEMBER	MAGNITUDE OF FORCE (in kN)	NATURE OF FORCE
1.	AC	2	COMPRESSION
2.	AB	1.7321	TENSION
3.	CB	0	-
4.	CE	2	COMPRESSION
5.	BE	1.1547	COMPRESSION
6.	BD	2.3094	TENSION
7.	DE	1.1547	COMPRESSION

8.	DG	1.7321	TENSION
9.	EF	2	TENSION
10.	EH	4	COMPRESSION
11.	FD	0	-
12	FG	2	COMPRESSION
13.	GH	0	-

Q.5(b) Determine the speed at which the basket ball at A must be thrown at an angle 30^o so that if makes it to the basket at B.

Also find at what speed it passes through the hoop. (6 marks)



Solution :

Given : $\theta=30^o$

To find : Speed at which basket ball must be thrown

Solution :

Assume that the basket ball be thrown with initial velocity u and it takes time t to reach B

HORIZONTAL MOTION

Here the velocity is constant

8 = ucos30 x t

$t = \frac{8}{ucos30} = \frac{9.2376}{u}$ ……..(1)

v_B = ucos30 (Since velocity is constant in horizontal motion) ………(2)

VERTICAL MOTION

Initial vertical velocity (u_v) = usin30 =0.5u …….(3)

Vertical displacement(s) = 2.4 - 1.2 = 1.2

$t = \frac{9.2376}{u}$

Using kinematical equation :

$s = ut + \frac{1}{2} \times at^2$

$1.2 = \frac{u}{2} \times \frac{9.2376}{u} - \frac{1}{2} \times 9.81 \times (\frac{9.2376}{u})^2$

u^2=122.4289

u=11.0648 m/s

u_v=0.5u (From 3)

u_v = 0.5 x 11.0648

= 5.5324 m/s

Using kinematical equation

$v_v^2 = u_v^2 + 2as$

$v_v^2 = 5.5324^2 - 2 \times 9.81 \times 1.2$

v_v = 2.6622 m/s

v_h = 11.0648cos30 = 9.5824 m/s (From 2)

$v_B = \sqrt{v_v^2 + v_h^2}$

v_B = 9.9441 m/s

$\alpha = \tan^{-1}(\frac{2.6577}{9.5824})$

$= 15.5015^o$

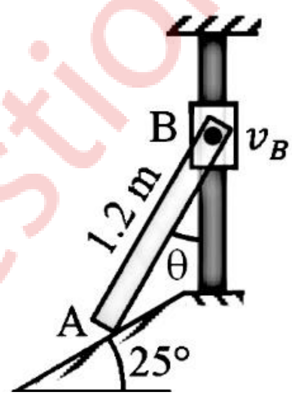
Speed at which the basket-ball at A must be thrown = 11.0648 m/s (30^o in first quadrant)

Speed at which the basket-ball passes through the hoop = 9.9441 m/s(15.5015^o in fourth quadrant)

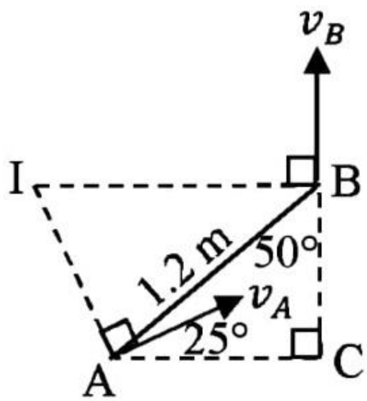
Q.5(c) Figure shows a collar B which moves upwards with constant velocity of 1.5 m/s.At the instant when θ=50°.Determine :

(i)The angular velocity of rod pinned at B and freely resting at A against 25o sloping ground.

(ii)The velocity of end A of the rod. (6 marks)



Solution:



ICR is shown in the given figure

BY USING GEOMETRY:

In △ABC

∠ABC = 50

∠ACB = 90

∠BAC = 40

∠CAV = 25

∠BAV = 40 – 25 = 15

IA ⊥ V_A

∠IAB = 90 – 15 = 75

∠IBA = 90 – 50 = 40

In △IBA

∠BIA = 180 – 75 = 65

In △IBA

AB=1.2 m

APPLYING SINE RULE

$$\frac{AB}{\sin I} = \frac{IB}{\sin A} = \frac{IA}{\sin B}$$

$$\frac{1.2}{\sin 65} = \frac{IB}{\sin 75} = \frac{IA}{\sin 40}$$

IB=1.2789 m

IA=0.8511 m

Assume ω_{AB} be the angular velocity of AB

$\omega_{AB} = \frac{v_B}{r} = \frac{v_B}{IB} = \frac{1.5}{1.2789} = 1.1728$ rad/s

v_A = r x AB = IA x ω_{AB} = 0.8511 x 1.7288 = 0.99825 m/s

Angular velocity of rod AB= 1.1728 rads (Anti-clockwise)

Instantaneous velocity of A = 0.9982 m/s(25^o in first quadrant)

Q.6(a) A force of 140 kN passes through point C (-6,2,2) and goes to point B (6,6,8). Calculate moment of force about origin. (4 marks)

Solution :

Given : C (-6,2,2)

B (6,6,8)

To find : Moment of force about origin

Solution :

Assume $\bar{b}$ and $\bar{c}$ be the position vectors of points B and C respectively w.r.t O (0,0,0)

$\overline{OB} = \bar{b} = 6\bar{\imath} + 6\bar{\jmath} + 8\bar{k}$

$\overline{OC} = -6\bar{\imath} + 2\bar{\jmath} + 2\bar{k}$

$\overline{CB} = (6\,\bar{\imath} + 6\bar{\jmath} + 8\bar{k}) - (-6\,\bar{\imath} + 2\bar{\jmath} + \bar{k})$

$= 2\,(6\,\bar{\imath} + 2\bar{\jmath} + 3\bar{k})$

$|\overline{CB}| = 2 \times \sqrt{6^2 + 2^2 + 3^2}$

$= 14$

Unit vector along $\overline{CB} = \frac{CB}{|CB|} = \frac{6i+2j+3k}{7}$

Force along $\overline{CB}$ = $\overline{F}$ = 140 x $\frac{6i+2j+3k}{7}$

= 120 $\bar{\imath}$ +40$\bar{\jmath}$+60$\bar{k}$

Moment of $\overline{F}$ about O = $\overline{OB}$ x $\overline{F}$

i	j	k
6	6	8
120	40	60

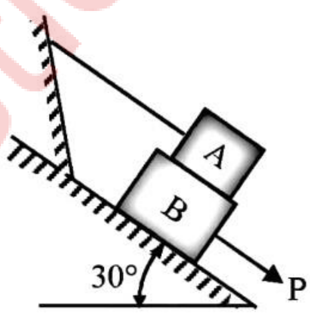
= 40 $\bar{\imath}$ + 600$\bar{\jmath}$ - 480k

Moment of F about C is 40 $\bar{\imath}$ + 600$\bar{\jmath}$ - 480$\bar{k}$ kNm

---

Q.6(b) Refer to figure.If the co-efficient of friction is 0.60 for all contact surfaces and θ = 30º,what force P applied to the block B acting down and parallel to the incline will start motion and what will be the tension in the cord parallel to inclined plane attached to A.

Take W_A=120 N and W_B=200 N. (8 marks)



Solution :

Given : : μ=0.6

θ = 30º

W_A = 120 N

W_B = 200 N

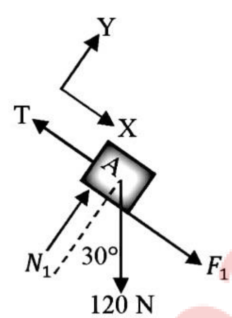
To find : Force P

Solution :

$F_1 = \mu N_1 = 0.6N_1$ ………(1)

$F_2 = \mu N_2 = 0.6N_2$ ……….(2)

Consider FBD of block A



The block is considered to be in equilibrium

Applying conditions of equilibrium

$\Sigma Fy = 0$

$N_1 - 120\cos 30 = 0$

N_1 = 103.923 N ……….(3)

From (1)

$F_1 = 0.6 \times 103.923$

$= 62.3538$ N

Applying conditions of equilibrium

$\Sigma Fx = 0$

$F_1 + 120\sin 30 - T = 0$

T = 122.3538 N

Consider FBD of block B

Applying conditions of equilibrium

$\Sigma Fy = 0$

$N_2 - N_1 - 200\cos30 = 0$

$N_2 = 277.1281$ N

$F_2 = 0.6 \times 277.1281$

$= 166.2769$ N From (2)

Applying conditions of equilibrium

$\Sigma Fx = 0$

$P - F_1 - F_2 + 200\sin30 = 0$

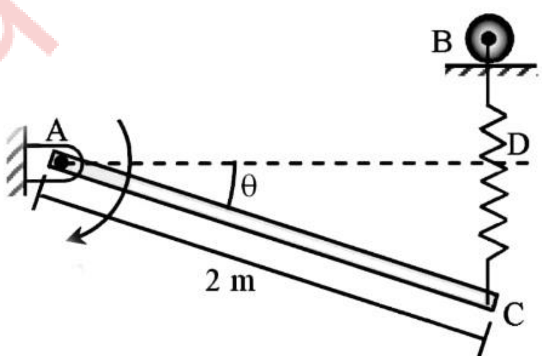
P = 128.6307 N

Force required on block B to start the motion is 128.6307 N

Tension T in the cord parallel to inclined plane attached to A=122.3538 N

Q.6(c) Determine the required stiffness k so that the uniform 7 kg bar AC is in equilibrium when $\theta = 30^o$.

Due to the collar guide at B the spring remains vertical and is unstretched when $\theta = 0^o$.Use principle of virtual work. (4 marks)



Solution:

Given : : Mass of bar AC = 7 kg

$\theta = 30^o$

To find : Required stiffness k

Solution:

Weight of rod = 7g N

Assume rod AC have a small virtual angular displacement $\delta\theta$ in anti-clockwise direction

Reaction forces H_A and V_A do not do any virtual work

Un-stretched length of the spring = BD

Extension of the spring (x) = CD = 2sin θ

Assume F_S be the spring force at end C of the rod

$F_S = Kx = 2K\sin\theta$

Assume A to be the origin and AD be the X-axis of the system

Active force	Co-ordinate of the point of action along the force	Virtual Displacement
W=7g	-sin θ	**δyM=-cos θ δ θ**
FS=2Ksin θ	-2sin θ	**δyC'=-2cos θ δ θ**

APPLYING PRINCIPLE OF VIRTUAL WORK

$\delta U = 0$

$-W \times \delta_{YM} + F_S \times \delta Y_{C'} + 50 \times \delta\theta = 0$

$2K\sin\theta \times (-2\cos\theta\,\delta\theta) = 7g \times (-\cos\theta\,\delta\theta) - 50 \times \delta\theta$

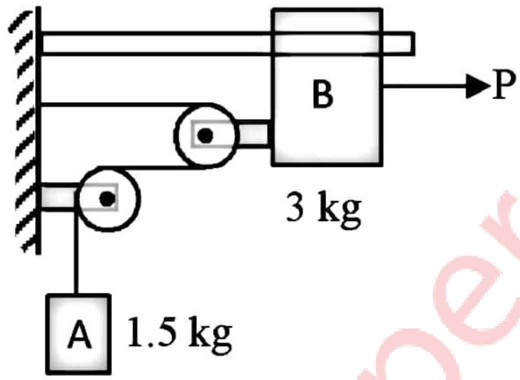
Substituting the value of θ and solving

K=63.2025 Nm

The required stiffness K for bar AC to remain in equilibrium is 63.2025 Nm

Q.6(d) The system in figure is initially at rest.

Neglecting friction determine the force P required if the velocity of the collar is 5 m/s after 2 sec and corresponding tension in the cable. (4 marks)



Solution :

For block B

u = 0

t = 2 s

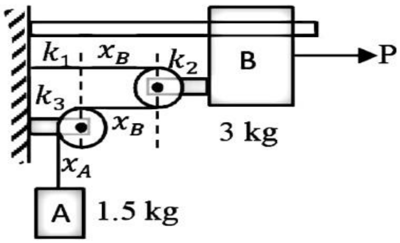
v = 5m/s

$a = \frac{5-0}{2} = 2.5\ m/s^2$ …………..(1)

Assume the string across the two pulleys be of length L

Assume x_A and x_B be the displacements of block A and collar B respectively

Assume k_1,k_2 and k_3 be the lengths of the string which remain constant irrespective of the position of block A and block B



$k_1 + x_B + k_2 + x_B + k_3 + x_A = L$

$\mathbf{x_A = L - k_1 - k_2 - k_3 - 2x_B}$

Differentiating with respect to time

$\mathbf{v_A = -2v_B}$

Differentiating with respect to time one again

$\mathbf{a_A = -2a_B}$

Considering only magnitude

$\mathbf{a_A = 2a_B}$

$a_A = 2 \times 2.5$

$= 5\ m/s^2$ ………..(2) (From 1)

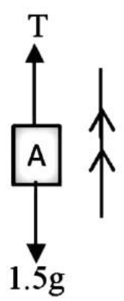
Weight of block A(WA) = $m_A g$

= 14.715 N

Assume T to be the tension in the string

Consider the vertical motion of block A

F.B.D of block A



$\Sigma Fy = m_A a_A$

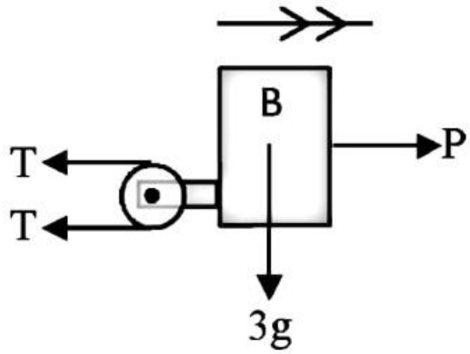
$T - W_A = m_A a_A$

$T - 14.715 = 1.5 \times 5$

$\mathbf{T = 22.215\ N}$ …………..(3)

Consider the horizontal motion of collar B

F.B.D of collar B



$\Sigma Fx = m_B a_B$

$P - 2T = m_B a_B$

$P - 2x22.215 = 3x2.5$

P = 51.93 N

Force P required = 51.93 N
Tension in the cable = 22.215 N

MUMBAI UNIVERSITY

SEMESTER -1

ENGINEERING MECHANICS QUESTION PAPER – DEC 2017

Q.1<u>Attempt any four questions</u>

Q.1(a) State and prove varigon's theorem.	(5 marks)

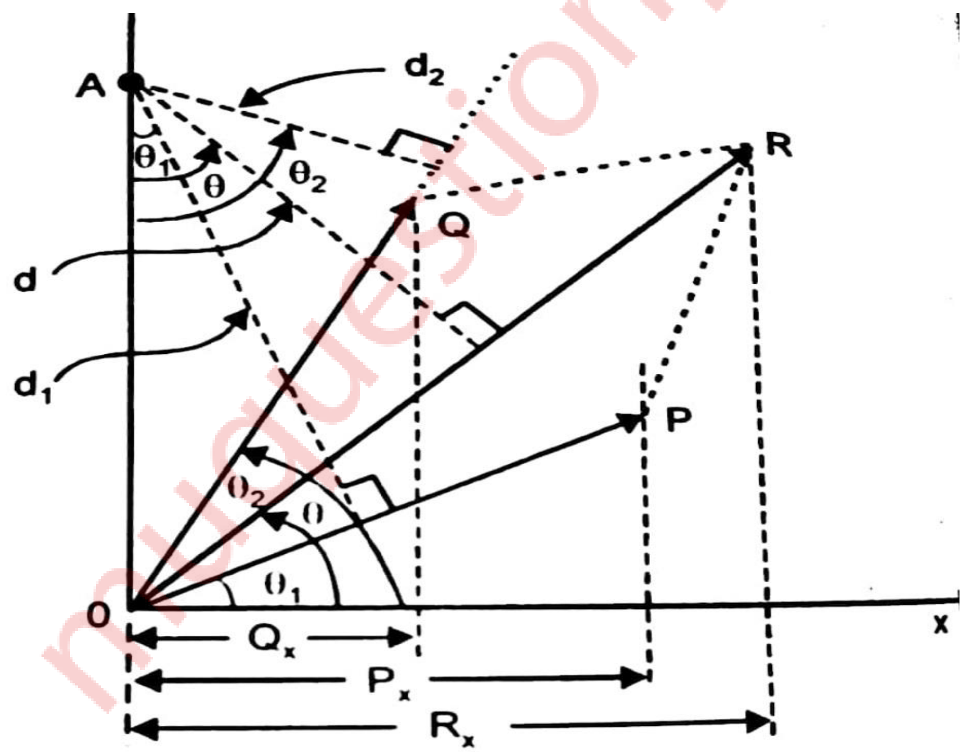
Solution:

Statement:

The algebraic sum of the moments of a system of coplanar forces about any point in the plane is equal to the moment of the resultant force of the system about the same point.

$\Sigma M_A^F = \Sigma M_A^R$

Proof:



Let P and Q be two concurrent forces at O,making angle θ_1 and θ_2 with the X-axis

Let R be the resultant making an angle θ with X axis

Let A be a point on the Y-axis about which we shall find the moments of P and Q and also of resultant R.

Let d_1,d_2 and d be the moment arm of P,Q and R from moment centre A

The x component of forces P,Q and R are P_x,Q_x and R_x

$\therefore M_A{}^P = P \times d_1$ ……..(1)

$\therefore M_A{}^Q = Q \times d_2$ ……..(2)

$\therefore M_A{}^R = R \times d$

$= R(OA.\cos\theta)$

$= OA.R_x$

Adding (1) and (2)

$\therefore M_A{}^P + M_A{}^Q = Pd_1 + Qd_2$

$\Sigma M_A{}^F = P \times OA\cos\theta_1 + Q \times OA\cos\theta_2$

$= OA.P_x + OA.Q_x$ (as $P_x = P.\cos\theta_1$ and $Q_x = Q\cos\theta_2$)

$= OA(P_x + Q_x)$

$\therefore \Sigma M_A{}^F = OA(R_x)$ ……..(3)

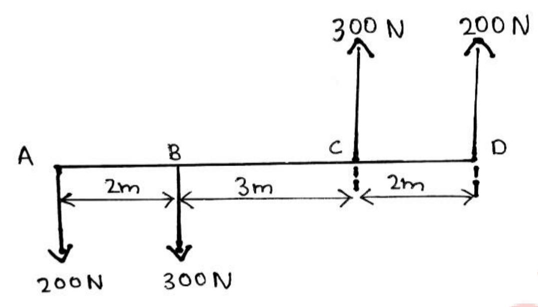
From (4) and (3)

$\Sigma M_A{}^F = \Sigma M_A$

Thus,Varigon's theorem is proved

Q.1(b) Find the resultant of the force system as shown in the given figure.

(5 marks)



Solution:

Taking forces having direction upwards as positive.

Net force = 200+300-200-300

=0 N

Taking moments of the forces about the point A

Taking anticlockwise moment direction as positive

∴ M_A = 200 x7 +300 x 5 – 300 x2

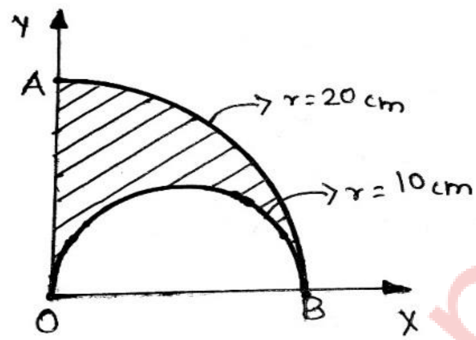
=2300 Nm (anticlockwise direction)

The resultant force is 0.

Net moment is 2300 Nm(anticlockwise)

Q.1(c) Find the co-ordinate of the centroid of the area as shown in the given figure.

(5 marks)



Solution:

Figure	Area(mm^2)	X co-ordinate of centroid (mm)	Y co-ordinate of centroid (mm)	A_x (mm^2)	A_y (mm^2)
Quarter circle	$0.25 \times \pi \times R^2$ $=0.25 \times 20^2 \times \pi$ $=314.1593$	$\frac{4R}{3\pi} = \frac{4 x 20}{3\pi}$ =8.4883	$\frac{4R}{3\pi} = \frac{4 x 20}{3\pi}$ =8.4883	2666.6667	2666.6667
Semi-circle (to be removed)	$-0.5 \times \pi \times r^2$ = -157.0796	10	$\frac{4R}{3\pi} = \frac{4 x 10}{3\pi}$ =4.2441	-1570.7963	-666.6667
Total	157.0796			1095.8704	2000

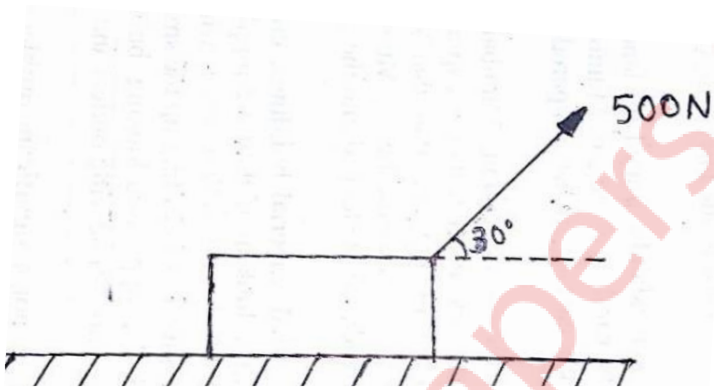
∴ X co-ordinate of centroid ($\overline{x}$)=$\frac{\Sigma Ax}{\Sigma A}$=$\frac{1095.8704}{157.0796}$ = 6.9765 cm

∴ Y co-ordinate of centroid ($\overline{y}$)=$\frac{\Sigma Ay}{\Sigma A}$=$\frac{2000}{157.0796}$ = 12.7324 cm

Centroid = (6.9765,12.7324) cm

Q.1(d) A force of 500 N is acting on a block of 50 kg mass resting on a horizontal surface as shown in the figure. Determine the velocity after the block has travelled a distance of 10m. Co efficient of kinetic friction is 0.5.

(5 marks)



Solution:

Given : Co-efficient of kinetic friction (μ_k)=0.5

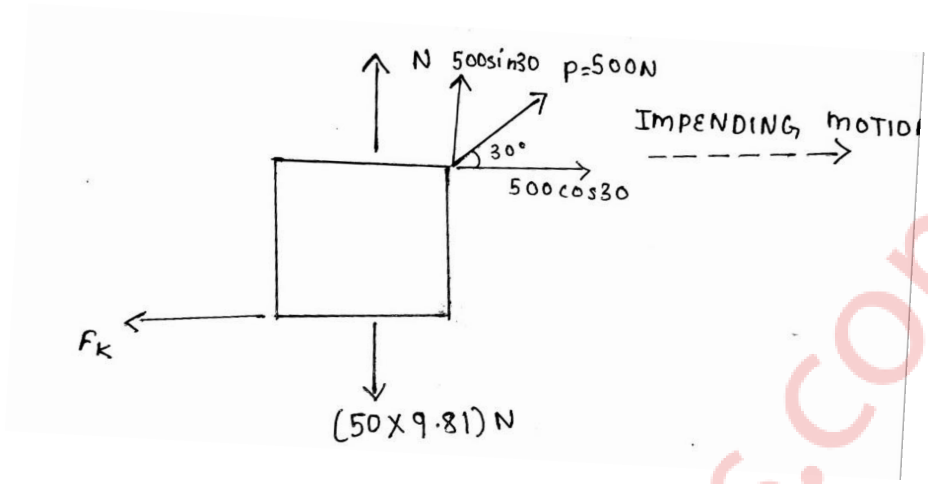
P = 500 N

m = 50 kg

u = 0 m/s

s = 10 m

To find : Velocity after the block has travelled a distance of 10 m



Solution:

The body has no motion in the vertical direction.

∴ $\Sigma F_y = 0$

∴ N - 50g + Psin30 = 0

∴N = 50g - 500sin30

Let us assume that F is the kinetic frictional force

∴ $F = \mu_k$ x N

∴F = 0.5(50 g – 500 sin 30)

∴F = 25g – 125

By Newton's second law of motion

$\sum F_x = ma$

∴Pcos Θ – F = 50a

∴50a = 312.7627

∴ a= 6.2553 m/s^2

By kinematics equation

$v^2 = u^2 + 2 \times a \times s$

∴$v^2 = 0^2 + 2 \times 6.2553 \times 10$

∴ v= 11.1851 m/s

The velocity of the block after travelling a distance of 10 m = 11.1851 m/s

Q.1(e) The position vector of a particle which moves in the X-Y plane is given by

$\bar{r}$ = $(3t^3-4t^2)\bar{\imath} + (0.5t^4)\bar{\jmath}$ (5 marks)

Solution:

Given : $\bar{r} = (3t^3-4t^2)\bar{\imath} + (0.5t^4)\bar{\jmath}$

To find : Velocity and acceleration at t=1s

Solution:

$\bar{r} = (3t^3-4t^2)\bar{\imath} + (0.5t^4)\bar{\jmath}$

Differentiating w.r.t to t

$\therefore \frac{d\bar{r}}{dt} = \bar{v} = (9t^2-8t)\ \bar{\imath} + (2t^3)$ m/s ……..(1)

Differentiating once again w.r.t to t

$\therefore \frac{d\bar{v}}{dt} = \bar{a} = (18t-8)\ \bar{\imath} + (6t^2)\ \bar{\jmath}$

$\therefore\ \bar{a} = (18t-8)\ \bar{\imath} + (6t2)\ \bar{\jmath}$ m/s^2 ………..(2)

At t = 1,

Substituting t=1 in (1) and (2)

At t=1 s

$\bar{v} = \bar{\imath} + 2\bar{\jmath}$ m/s

$\bar{a} = 10\bar{\imath} + 6\bar{\jmath}$ m/s^2

For magnitude :

v = $\sqrt{1^2 + 2^2}$

$=\sqrt{5}$

$=2.2361$ m/s

$a = \sqrt{10^2 + 6^2}$

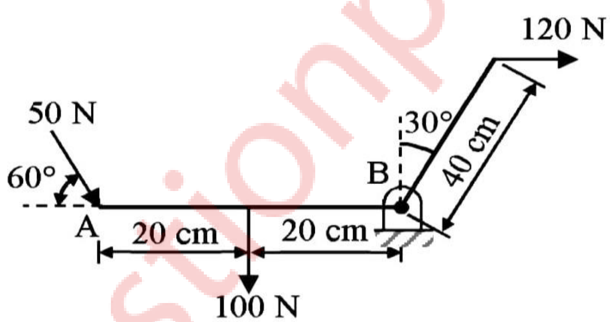
$= \sqrt{136}$

$= 11.6619$ m/s^2

Velocity at t=1s is 2.2361 m/s

Acceleration at t=1s is 11.6619 m/s^2

Q 2 a) Find the resultant of the force acting on the bell crank lever shown. Also locate its position with respect to hinge B. (8 marks)



Given : Forces on the bell crank lever

To find : Resultant and it's position w.r.t hinge B

Solution:

Let the resultant of the system of forces be R and it is inclined at an angle θ to the horizontal

The hinge is in equilibrium

Taking direction of forces towards right as positive and towards upwards as positive

Applying the conditions of equilibrium

$\Sigma F_x = 0$

$R_x = 50\cos 60 + 120$

$= 145$ N

$R_y = -50\sin 60 - 100$

$= -143.3013$

$R = \sqrt{R_x^2 + R_y^2}$

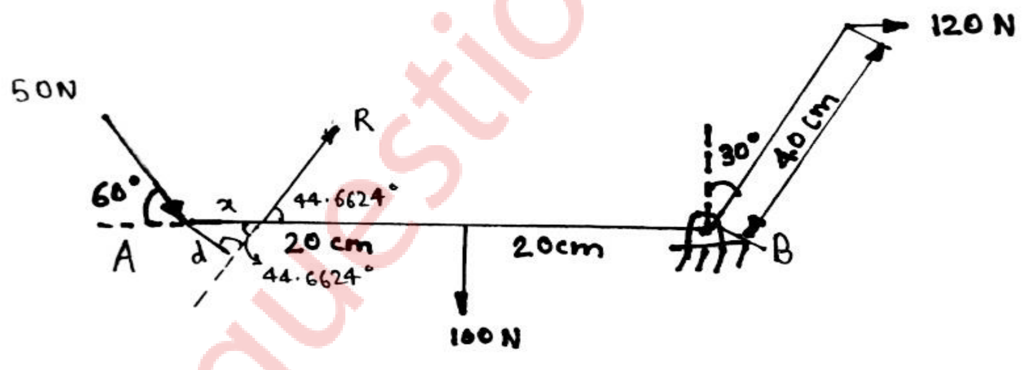
$= \sqrt{145^2 + (-143.3013)^2}$

$= 203.8633$ N

$\theta = \tan^{-1}(\frac{R_y}{R_x})$

$= \tan^{-1}(\frac{143.3013}{145})$

$= 44.6624^o$



Let the resultant force R be acting at a point x from the point A and it is at a perpendicular distance of d from point A

Taking moment of forces about point A and anticlockwise moment as positive

Applying Varigon's theorem,

203.8633 x d = -(100 x 20) – (120 x 40cos30)

d = -30.2012 cm = 30.2012 cm ……….(as distance is always positive)

sin 44.6624 = $\frac{x}{30.2012}$

x = 21.2293 cm

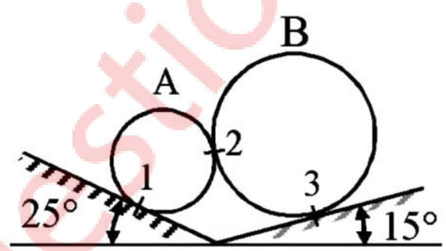
Distance from point B = 40 – 21.2293

=18.7707 cm

Resultant force = 203.8633 N (at an angle of 44.6624° in first quadrant)

Distance of resultant force from hinge B = 18.7707 cm

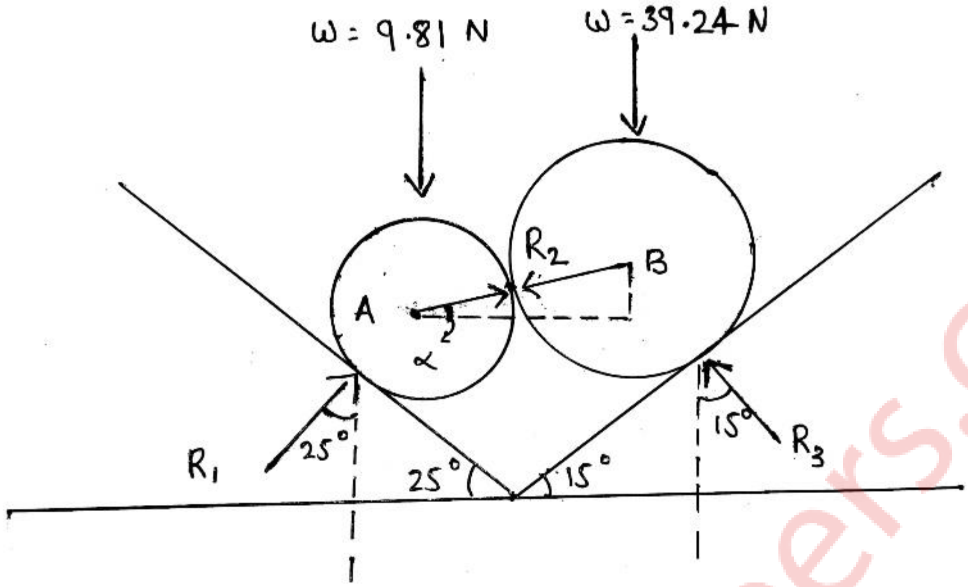
Q2b) Determine the reaction at points of constant 1,2 and 3. Assume smooth surfaces.



(6 marks)

Given: The spheres are in equilibrium

To find: Reactions at points 1,2 and 3



Solution:

Considering both the spheres as a single body

The system of two spheres is in equilibrium

Applying conditions of equilibrium:

$\sum F_y=0$

$R_1\cos25 + R_3\cos15 – g - 4g = 0$

$R_1\cos25 + R_3\cos15 = 5g$ ……(1)

$\sum F_x = 0$

$R_1\sin25 - R_3\sin15 = 0$ …….(2)

Solving (1) and (2)

$R_1 = 19.75$ N and $R_2 = 32.2493$ N …….(3)

Let the reaction force between the wo spheres be R_2 and it acts at an angle α with X-axis

Sphere A is in equilibrium

Applying conditions of equilibrium

$\sum F_y=0$

$R_1\cos25 - R_2\sin\alpha – g = 0$

$R_2 sin\alpha = 8.0896$ ……..(4) (From 3)

$\sum F_x=0$

$R_1 sin25 - R_2 cos\alpha = 0$

$R_2 cos\alpha = 19.75 sin25$

$R_2 cos\alpha = 8.3467$ ………(5)

Squaring and adding (4) and (5)

$R_2^2(cos^2\alpha+sin^2\alpha) = 135.1095$

$R_2=11.6237$ N

Dividing (4) by (5)

$$\frac{R_2 sin\alpha}{R_2 cos\alpha}=\frac{8.0896}{8.3467}$$

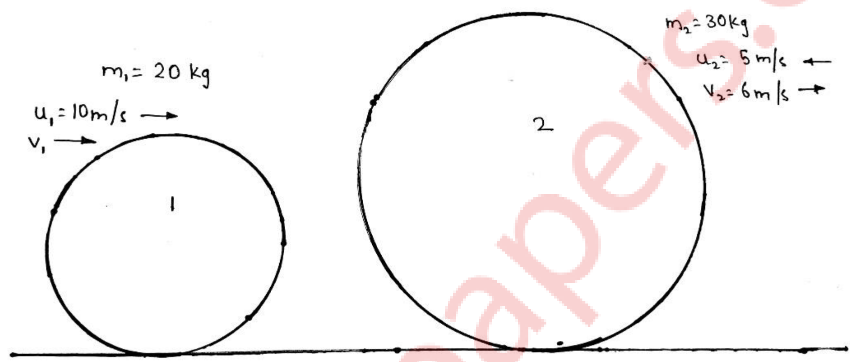
$\alpha = tan^{-1}(0.9692)$

$=44.1038^o$

R_1=19.75 N (75^o with positive direction of X-axis in first quadrant)
R_2=11.6237 N (44.1038^o with negative direction of X-axis in third quadrant)
R_3=32.2493 N (75^o with negative direction of X axis in second quadrant)

Q.2 c) Two balls having 20kg and 30 kg masses are moving towards each other with velocities of 10 m/s and 5 m/s respectively as shown in the figure.

If after the impact ,the ball having 30 kg mass is moving with 6 m/s velocity to the right then determine the coefficient of restitution between the two balls. (6 marks)



Solution:

Taking direction of velocity towards right(→) as positive and vice versa

Given : m_1=20 kg

m_2=30 kg

Initial velocity of ball $m_1(u_1)$=10 m/s

Initial velocity of ball $m_2(u_2)$= -5 m/s

Final velocity of ball $m_2(v_2)$ = 6 m/s

To find : Co-efficient of restitution(e)

Solution:

This is a case of direct impact as the centre of mass of both balls lie along a same line.

According to the law of conservation of momentum:

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$\therefore$ 20 x 10 + 30 x (-5) =20 x v1 +30 x 6

$\therefore$ 200 – 150 =20 x v1 + 180

$\therefore$ -130 = 20 x v1

$\therefore$ v1 = -6.5 m/s

Co-efficient of restitution (e) = $(v2 - v1)/(u1 - u2)$

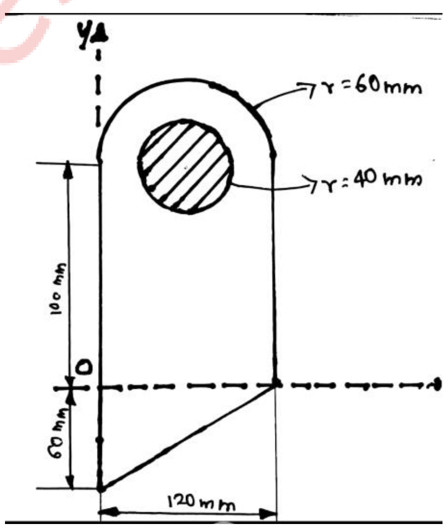
$\therefore$ e = (6 – (-6.5))/(10-(-5))

$\therefore$e=12.5/15

$\therefore$e=0.8333

The co-efficient of restitution (e) between the two balls is 0.8333

Q.3(a) Determine the position of the centroid of the plane lamina. Shaded portion is removed. (8 marks)



Solution:

FIGURE	AREA (mm^2)	X co-ordinate Of centroid (mm)	Y co-ordinate Of centroid (mm)	A_x (mm^2)	A_y (mm^2)
Rectangle	120 x 100 =12000	$\frac{120}{2} = 60$	$\frac{120}{2} = 60$	720000	600000
Triangle	$\frac{1}{2}$ x 120 x 60 =3600	$\frac{120}{3} = 40$	$\frac{-60}{3} = -20$	144000	-72000
Semicircle	$\frac{1}{2}$ x π x 60^2 =1800 π =5654.8668	$\frac{120}{2} = 60$	$100 + \frac{4*60}{3\pi}$ =125.4648	339292.01	709486.68
Circle (Removed)	- π x 40^2 =5026.5482	$\frac{120}{2} = 60$	100	-301592.89	-502654.82
Total	16228.32			901699.12	734831.86

$$\frac{\Sigma Ax}{\Sigma A} = \frac{901699.12}{16228.32} = 55.56 \text{ mm}$$

$$\frac{\Sigma Ay}{\Sigma A} = \frac{734831.86}{16228.32} = 45.28\ mm$$

Centroid is at (55.56,45.28)mm

Q3(b) Explain the conditions for equilibrium of forces in space. (6 marks)

Answer:

A body is said to be in equilibrium if the resultant force and the resultant momentum acting on a body is zero.

For a body in space to remain in equilibrium, following conditions must be satisfied:

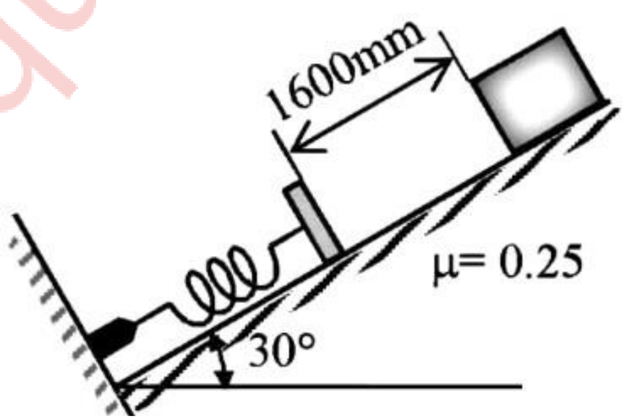
(1) Algebraic sum of the X components of all the forces is zero.
$\Sigma F_x = 0$

(2)Algebraic sum of the Y components of all the forces is zero.
$\Sigma F_y = 0$

(3)Algebraic sum of the Z components of all the forces is zero.
$\Sigma F_z = 0$

(4)Algebraic sum of the moment of all the forces about any point in the space is zero.

Q.3(c) A 30 kg block is released from rest.If it slides down from a rough incline which is having co-efficient of friction 0.25.Determine the maximum compression of the spring.Take k=1000 N/m. (6 marks)



Solution:

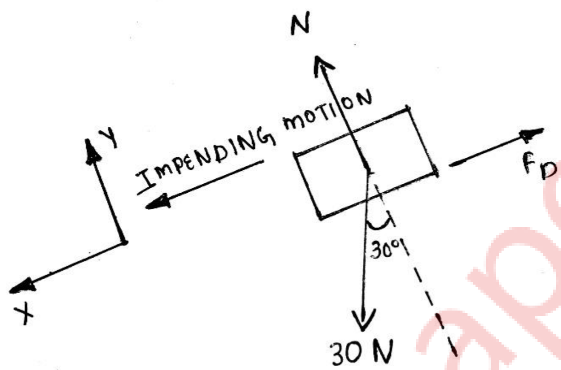
Given : Value of spring constant = 1000 N/m

W = 30N

μs = 0.25

To find : Maximum compression of the spring

Solution :



Let the spring be compressed by x cm when the box stops sliding

N = Wcos30

= 30 x 0.866

= 25.9808 N

Frictional force = μ_sN

= 0.25 x 25.9808

= 6.4952 N

Displacement of block = (1.6+x) m

Work done against frictional force = F_D x s

=6.4952(1.6+x)

At position 1

v_1=0 m/s

Vertical height above position(II) = h = (1.6+x) sin30

PE_1 = mgh = 30(1.6+x)sin30 = 15(1.6+x)

$KE_1 = \frac{1}{2}$ x mv_1^2=0

Compression of spring=0

Initial spring energy = $\frac{1}{2}$ x K x^2 = 0

At position II

Assuming this position as ground position

H^2 = 0

$P.E^2$ = 0

Speed of block v = 0

$K.E_2 = \frac{1}{2}$ x mv^2 = 0

Compression of spring = x

Final spring energy = $E_S = \frac{1}{2}$x K x (x^2)

= 0.5 x 1000 x x^2

= $500x^2$

Appling work energy principle for the position (I) and (II)

$U_{1\text{-}2} = KE_2 - KE_1$

$-W_F + PE_1 - PE_2 - E_S = KE_2 - KE_1$

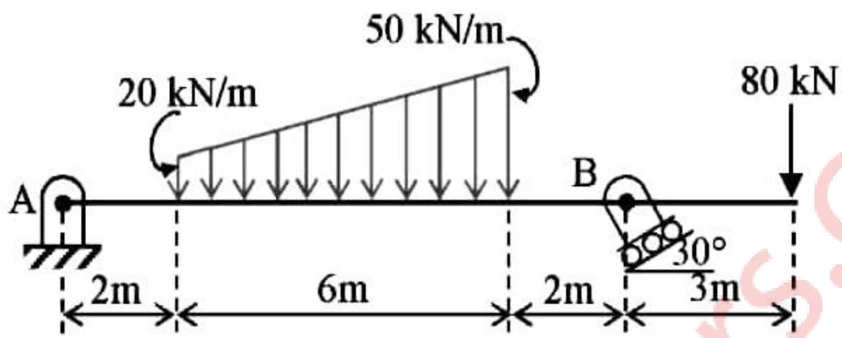
-6.4952(1.6+x) + 15(1.6+x)- 0- 500 x 2 = 0-0

$500x^2$ - 8.5048x - 13.6077= 0

x=0.1737 m

The maximum compression of the spring is 0.1737 m

Q.4(a)Find the support reactions at A and B for the beam loaded as shown in the given figure. **(8 marks)**



Solution:

Given : Various forces on beam

To find : Support reactions at A and B

Solution:

Draw PQ ⊥to RS

Effective force of uniform load =20 x 6 = 120 kN

$2 + \frac{6}{2} = 5$ m

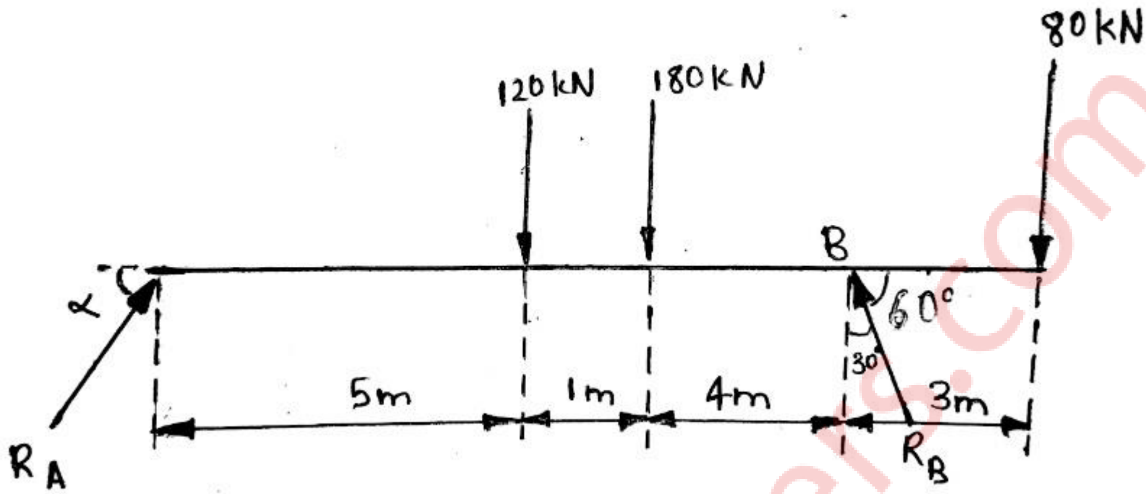
This load acts at 5m from A

Effective force of uniformly varying load $=\frac{1}{2}$ x (80-20) x 6

=180 kN

$2+\frac{6}{3}$ x 2 =6m

This load acts at 6m from A



The beam is in equilibrium

Applying the conditions of equilibrium

$\sum M_A = 0$

$-120 \times 5 - 180 \times 6 + R_B \cos 30 \times 10 - 80 \times 13 = 0$

$10R_B \cos 30 = 120 \times 5 + 180 \times 6 + 80 \times 13$

RB = 314.0785 N

Reaction at B will be at 60^o in second quadrant

$\sum F_x = 0$

$R_A \cos\alpha - R_B \sin 30 = 0$

$R_A \cos\alpha - 314.0785 \times 0.5 = 0$

$R_A \cos\alpha = 157.0393$ N ……..(1)

$\sum Fy = 0$

$R_A \sin\alpha - 120 - 180 + RB\cos 30 - 80 = 0$

$R_A \sin\alpha = 12 + 180 - 314.0785 \times 0.866 + 80$

$R_A \sin\alpha = 108.008$N ……..(2)

Squaring and adding (1) and (2)

$R_A^2(\sin^2\alpha + \cos^2\alpha) = 36325.3333$

$R_A = 190.5921$ N

Dividing (2) by (1)

$\frac{R_A sin\alpha}{R_A cos\alpha} = \frac{108.008}{157.0393}$

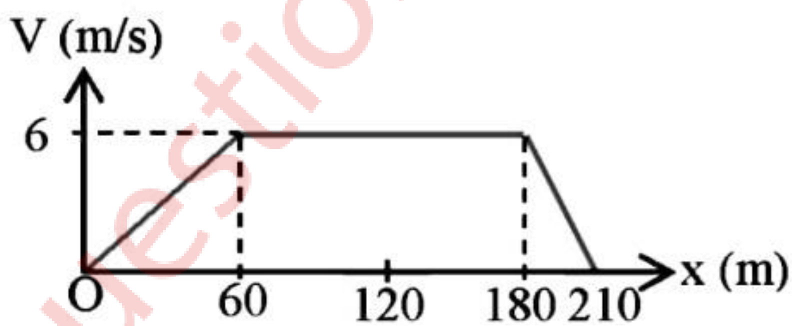
$\alpha = \tan^{-1}(0.6877)$

$= 34.5173^o$

Reaction at point A = 190.5921 N at 34.5173° in first quadrant

Reaction at B = 314.0785 N at 60° in second quadrant

Q 4b) The V-X graph of a rectilinear moving particle is shown. Find the acceleration of the particle at 20m,80 m and 200 m. (6 marks)



Solution :

Given : V-X graph of a rectilinear moving particle

To find : Acceleration of the particle at 20m,80 m and 200 m.

Solution :

$a = v\frac{dv}{dx}$

Part 1: Motion from O to A

O is (0,0) and A is (60,6)

Slope of v-x curve $\frac{dv}{dx}=\frac{6-0}{60-0}=0.1s^{-1}$

Average velocity = $\frac{u+v}{2} = \frac{6+0}{2}$ = 3 m/s

$a_{OA} = v\frac{dv}{dx}$ = 3 x 0.1 = 0.3 m/s^2

Part 2: Motion from A to B

A is (60,6) and B is (180,6)

$\frac{dv}{dx} = \frac{6-6}{180-60} = 0$ m/s^2

$a_{AB} = v\frac{dv}{dx}$ = 0 m/s^2

Part 3: Motion from B to C

B is (180,6) and C is (210,0)

$\frac{dv}{dx}=\frac{0-6}{210-180}$ = -0.2s^{-1}

Average velocity = $\frac{u+v}{2} = \frac{6+0}{2}$ = 3 m/s

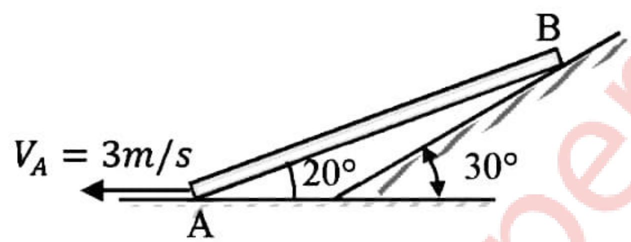
$a_{BC} = v\frac{dv}{dx}$ = 3 x (-0.2) = -0.6 m/s^2

Acceleration of particle at x = 20 m is 0.3m/s^2

Acceleration of particle at x = 80 m is 0 m/s^2

Acceleration of particle at x = 200 m is -0.6 m/s^2

Q.4(c) A bar 2 m long slides down the plane as shown.The end A slides on the horizontal floor with a velocity of 3 m/s.Determine the angular velocity of rod AB and the velocity of end B for the position shown. (6 marks)



Solution:

Given : $v_a = 3$ m/s

Length of bar AB = 2 m

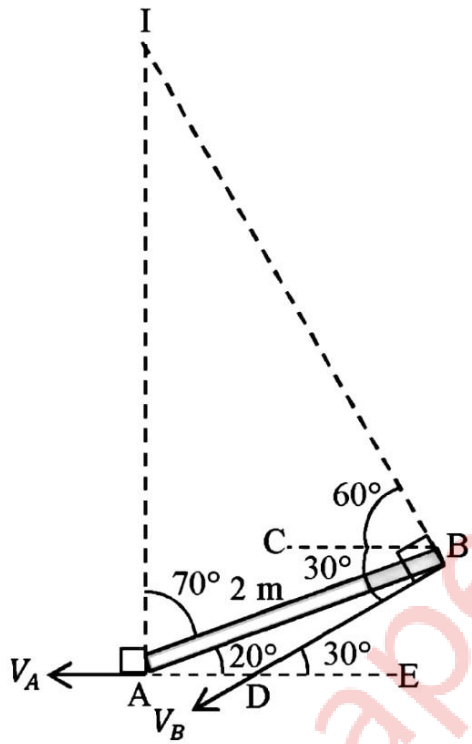
To find : Angular velocity ω

Velocity of end B

Solution:

Let ω be the angular velocity of the rod AB

ICR is shown in the free body diagram



Using Geometry:

∠BDE=30^o, ∠BAD=20^o

∠CBD= ∠BDE=30^o

∠CBA= ∠BAD=20^o

∠CBI=90^o-30^o=60^o

∠ABI= ∠CBI + ∠CBA=60^o+20^o=80^o

∠BAI=90^o-20^o=70^o

In △IAB, ∠AIB=180^o-80^o-70^o=30^o

By sine rule, $\frac{AB}{\sin I} = \frac{IB}{\sin A} = \frac{IA}{\sin B}$

$\therefore \frac{2}{sin 30} = \frac{IB}{\sin 70} = \frac{IA}{\sin 80}$

$\therefore$ IB = $\frac{2 sin 70}{\sin 30}$ = 3.7588 m

∴ IA = $\frac{2 \sin 80}{\sin 30}$ = 3.9392 m

∴ Angular velocity of the rod AB = $\frac{va}{r}$ = $\frac{3}{3.9392}$ = 0.7616 rad/s (clockwise direction)

∴ Instantaneous velocity of point B = rω = IB x ω = 3.7588 x 0.7616 = 2.8626 m/s

The instantaneous velocity at point B is always inclined at 30° in the third quadrant (as shown in the free body diagram)

Angular velocity of the rod AB = 0.7616 rad/s (clockwise)

Instantaneous velocity at point B = 2.8626 m/s (30° ↙)

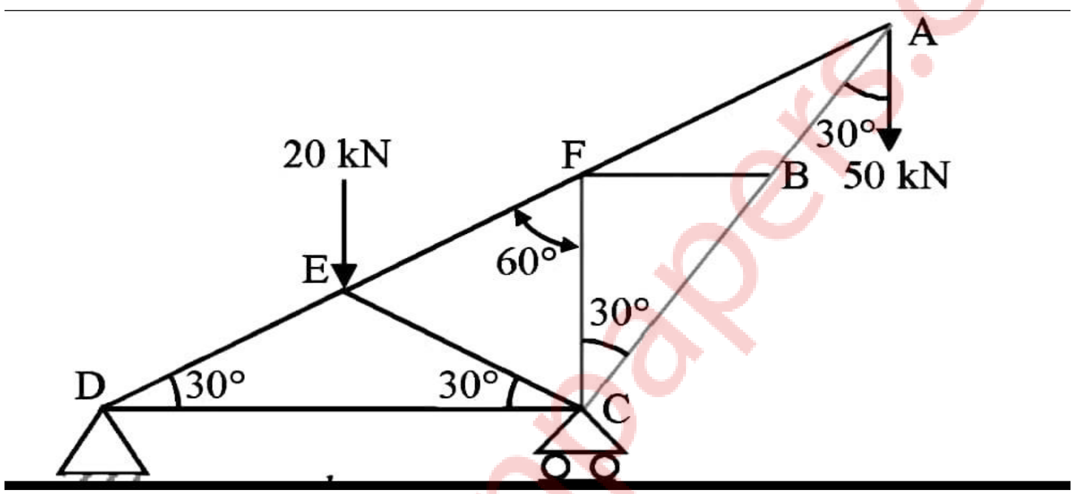
Q.5(a)Referring to the truss shown in the figure. Find :

(a) Reaction at D and C

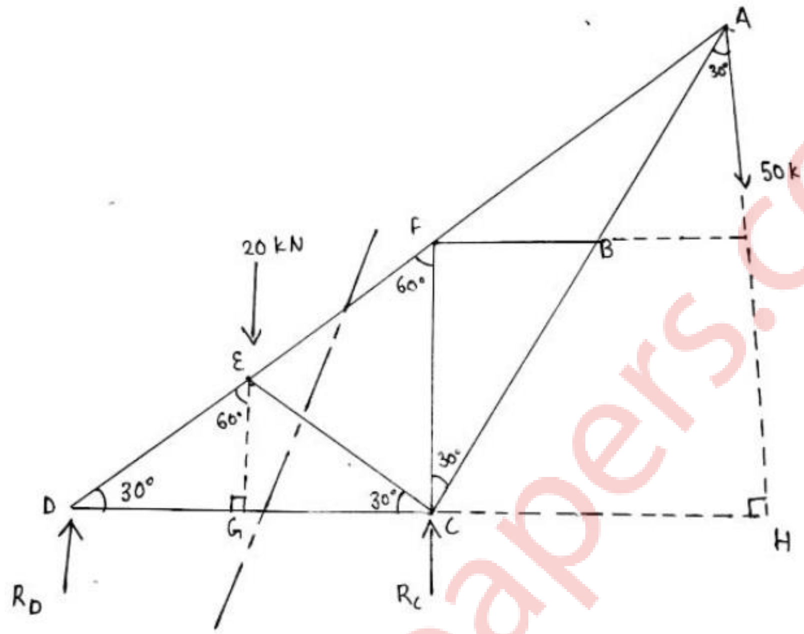
(b)Zero force members.

(c)Forces in member FE and DC by method of section.

(d)Forces in other members by method of joints. (8 marks)



Solution:



By Geometry:

In Δ ADC, ∠ADC = ∠CAD = 30°

AC = CD = l

Similarly, in Δ EDC,

ED = EC

Δ DEG and Δ CEG are congruent

DG = GC = $\frac{l}{2}$

In Δ DEG, ∠EDG=30°, ∠DGE=90°

tan30 = $\frac{EG}{DG}$

EG = DG.tan30 = $\frac{l}{2}$ x $\frac{1}{\sqrt{3}}$ = $\frac{l}{2\sqrt{3}}$

In Δ ACH,

CH = $\frac{AC}{2}$ = $\frac{l}{2}$

$DH = DC + CH = l + \frac{l}{2} = \frac{3l}{2}$

No horizontal force is acting on the truss, so no horizontal reaction will be present at point A

The truss is in equilibrium

Applying the conditions of equilibrium

$\Sigma M_D = 0$

-20 x DG -50 x DH + RC x DC = 0

$-20 \times \frac{l}{2} - 50 \times \frac{3l}{2} + RC \times l = 0$

-10 - 75 + RC = 0

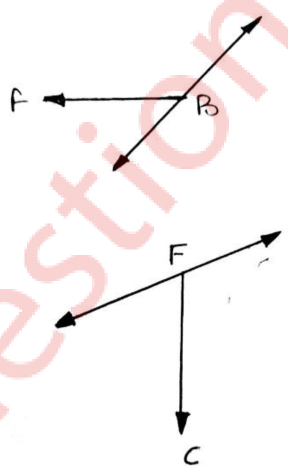
$R_C = 85$ kN

$\Sigma F_y = 0$

$-20 - 50 + R_D + R_C = 0$

$R_D = -15$kN

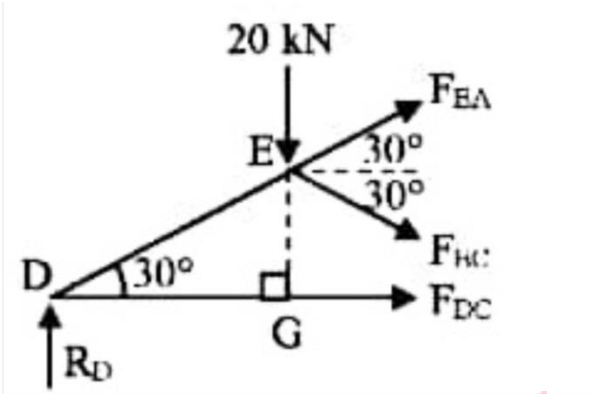
Loading at point B and F is shown



As per the rule,member BF will have zero force and is a zero force number.

Similarly,Member CF will have zero force

Method of sections :



Applying the conditions of equilibrium to the section shown

$\Sigma M_D = 0$

$-20 \times DG - F_{EC}\cos 30 \times EG - F_{EC}\sin 30 \times DG = 0$

$-20 \times \frac{l}{2} \times - F_{EC}\cos 30 \times EG - F_{EC}\sin 30 \times DG = 0$

$-20 \times \frac{l}{2} \times -F_{EC} \times \frac{\sqrt{3}}{2} \times \frac{l}{2} -F_{EC} \times \frac{1}{2} \times \frac{l}{2} = 0$

$-10 \times l - F_{EC} \times \frac{l}{4} - F_{EC} \times \frac{l}{4} = 0$

$-\frac{2l}{4} F_{EC} = 10L$

$F_{EC} = -20kN$

$R_D - 20 - F_{EC}\sin 30 + F_{EA}\sin 30 = 0$

$-15 - 20 + 20 \times 0.5 + F_{EA} \times 0.5 = 0$

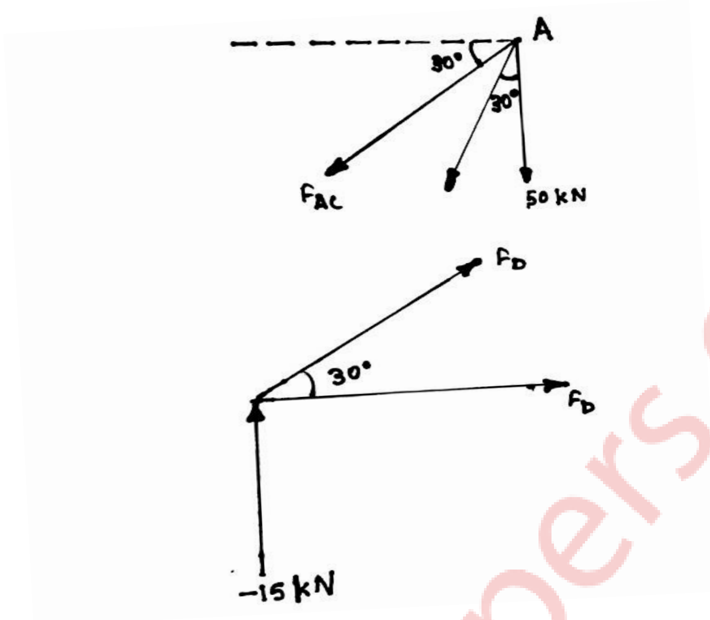
$F_{EA} = 50kN$

$F_{EC}\cos 30 + F_{EA}\cos 30 + F_{DC} = 0$

$-20 \times 0.866 + 50 \times 0.866 + F_{DC} = 0$

$F_{DC} = -25.9808kN$

Method of joints:



Joint A

$-50 - F_{AE}\sin30 - F_{AC}\cos30 = 0$

$-50 - 50 \times 0.5 = F_{AC} \times 0.866$

$F_{AC} = -86.6025kN$

Joint D

$F_{DC} + F_{DE}\cos30 = 0$

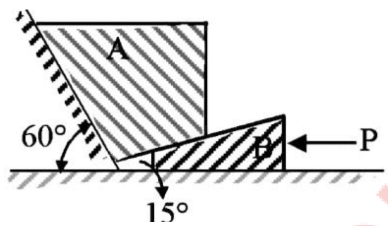
$-25.9808 + 0.866F_{DE} = 0$

$F_{DE} = 30kN$

Final answer :

Member	Magnitude (in kN)	Nature
AE (AF and EF)	50	Tension
AC (AB and BC)	86.6025	Compression
EC	20	Compression
DE	30	Tension
DC	25.9808	Compression
FB	0	
FC	0	

Q.5b) Determine the force P required to move the block A of 5000 N weight up the inclined plane, coefficient of friction between all contact surfaces is 0.25. Neglect the weight of the wedge and the wedge angle is 15 degrees. (6 marks)

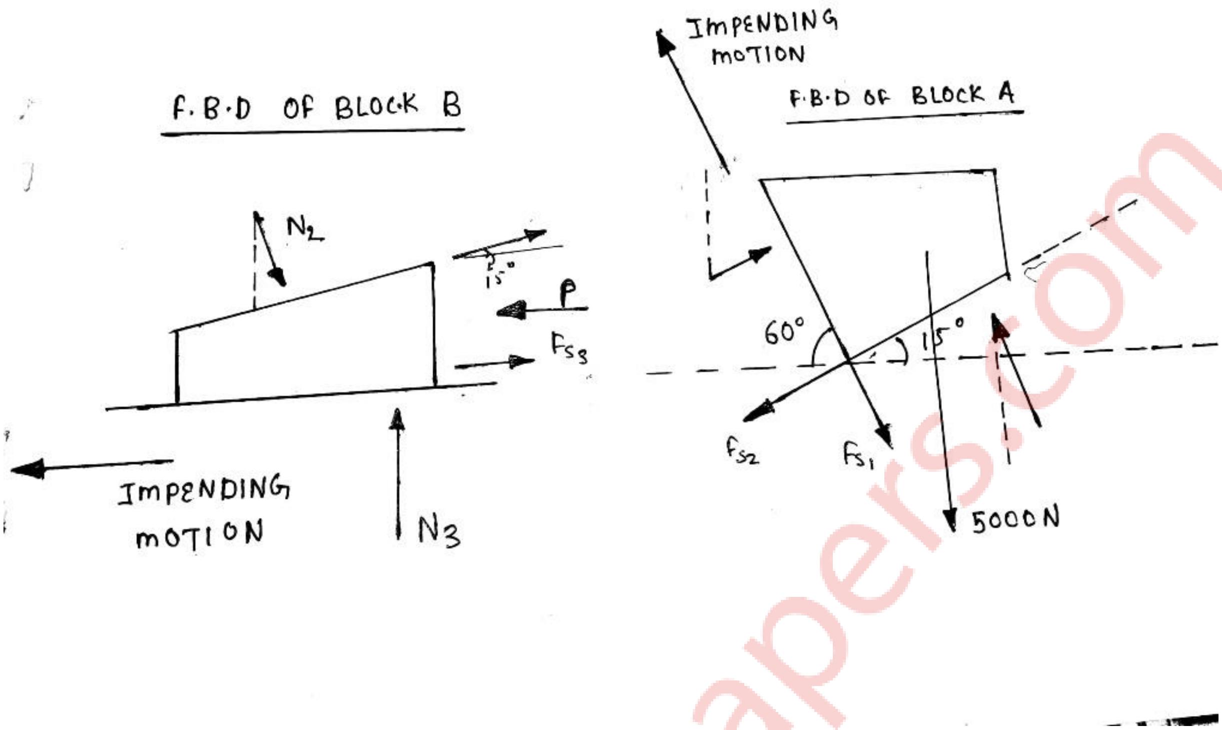


Given : Weight of block A = 5000 N

μ_s=0.25

Wedge angle = 15°

To find : Force P required to move block A up the inclined plane



Solution:

The impending motion of block A is to move up

The block A is in equilibrium

N_1,N_2,N_3 are the normal reactions

$F_{s1} = \mu_1 N_1 = 0.25N_1$

$F_{s2} = \mu_2 N_2 = 0.25N_2$

$F_{s3} = \mu_3 N_3 = 0.25N_3$

Applying the conditions of equilibrium

$\Sigma F_y = 0$

$\therefore$ -5000 + N_1 cos 60 – F_{s1} sin 60 – F_{s2} sin 15 + N_2 cos 15 = 0

$\therefore$ N_1 x 0.5 – 0.25N_1 x 0.866 -0.25 N_2 x 0.2588 + N_2 x 0.9659 = 5000 (From 1)

$\therefore$ 0.2835 N_1 +0.9012 N_2 = 5000 ……………..(2)

Applying the conditions of equilibrium

$\Sigma F_x = 0$

$\therefore N_1 \sin 60 + F_{s1} \cos 60 - F_{s2} \cos 15 - N_2 \sin 15 = 0$

$\therefore 0.866\ N_1 + 0.25 \times N_1 \times 0.5 - 0.25 \times N_2 \times 0.9659 - N_2 \times 0.2588 = 0$(From 1)

$\therefore 0.991\ N_1 - 0.5003\ N_2 = 0$

Solving equation, no 2 and 3

$N_1 = 2417.0851$ N

$N_2 = 4787.79$ N

The impending motion of block B is towards left

Block B is in equilibrium. Applying the conditions of equilibrium

$\Sigma F_y = 0$

$\therefore N_3 + F_{s2} \sin 15 - N_2 \cos 15 = 0$

$\therefore N_3 + 0.25 N_2 \times 0.2588 - N_2 \times 0.9659 = 0$

$\therefore N_3 - 0.9012\ N_2 = 0$

$\therefore N_3 = 0.9012 \times 4787.79 = 4314.7563$

Applying conditions of equilibrium

$\Sigma F_x = 0$

$\therefore -P + F_{s3} + F_{s2} \cos 15 + N_2 \sin 5 = 0$

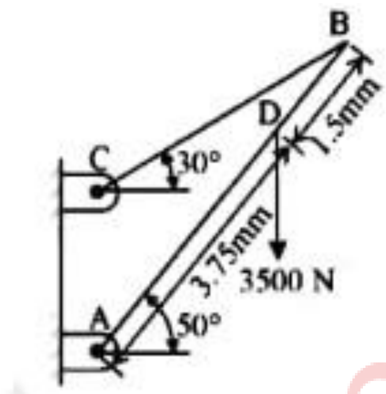
$\therefore 0.25\ N_3 + 0.25\ N_2 \times 0.9659 + N_2 \times 0.2588 = P$

$\therefore P = 0.25\ N_3 + 0.5003\ N_2 = 0.25 \text{ X } 4314.7563 + 0.5003 \times 4787.79 = 3474$ N

The force P required to move the block A of weight 5000 N up the inclined plane is P=3474 N

Q 5c) Determine the tension in a cable BC shown in fig by virtual work method.

(6 marks)



Given: F=3500 N

ϴ = 50o

Length of rod = 3.75 mm + 1.5 mm = 5.25 mm

To find : Tension in cable BC

Solution:

Let rod AB have a small virtual angular displacement θ in the clockwise direction

No virtual work will be done by the reaction force RA since it is not an active force

Assuming weight of rod to be negligible

Let A be the origin and dotted line through A be the X-axis of the system

Active force(N)	Co-ordinate of the point of action along the force	Virtual displacement
3500	Y co-ordinate of D=y_D=3.75sin θ	δy_D=3.75cos θδθ
Tcos30	X co-ordinate of B=x_B=5.25cos θ	δx_B=-5.25sin θδθ

Tsin30	Y co-ordinate of B=y_B=5.25sin θ	δy_B=5.25cos θδθ

By principle of virtual work :

-3500 x yd -Tsin 30 x yB -Tcos 30 x xB =0

-3500(3.75cos θδθ) – Tsin30(5.25cos θδθ) -Tcos30(-5.25sin θ δθ) = 0

Putting value of θ = 50° and dividing the above equation by δθ

(-3500 x 3.75cos50) – (Tsin30 x 5.25 cos50) + (Tcos30 x 5.25 sin50) = 0

5.25T(-sin30.cos50 + cos30.sin 50) = 3500 x 3.75 cos 50

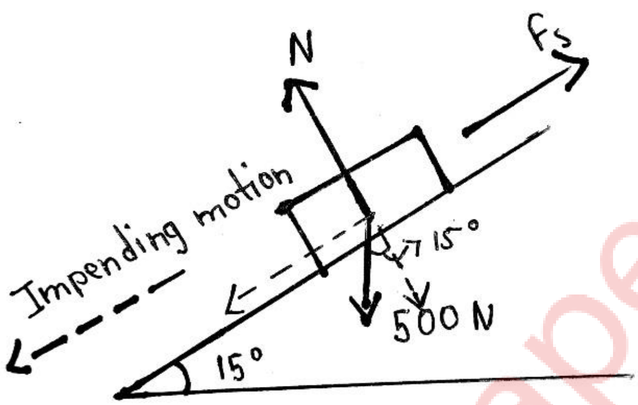
$$T = \frac{3500 * 3.75cos50}{5.25(\cos 30.sin50 - sin30.cos50}$$

$$= \frac{3500 * 3.75cos50}{5.25sin20}$$

= 4698.4631 N

The tension in the cable BC is 4698.4631 N

Q 6a) A 500 N Crate kept on the top of a 15° sloping surface is pushed down the plane with an initial velocity of 20m/s. If μs = 0.5 and μk = 0.4, determine the distance travelled by the block and the time it will take as it comes to rest. (5 marks)



Given: Weight of crate = 500 N

Initial velocity(u) = 20 m/s

μs = 0.5

μk = 0.4

θ = 15°

Final velocity (v) = 0 m/s

To find: Distance travelled by the block

Time it will take before coming to rest

Solution:

Mass (M) = $\frac{W}{g}$

$= \frac{500}{9.81}$

=50.9684 kg

Normal reaction (N) on the crate = 500 cos 15

Kinetic friction (F_k) = μ_k x N

= 0.4 x 500cos 15

= 193.1852 N

Let T be the force down the incline

Taking forces towards right of the crate as positive and forces towards left as negative

T + F_k = 500sin15

∴ T = 500sin15 – 193.1852

∴ T = -63.7756 N

By Newton's second law of motion

a = F/m

∴ a = $\frac{-63.7756}{50.9684}$ = -1.2513 m/s^2

Using kinematical equation:

$v^2 = u^2 + 2as$

∴ 0 = 202 – 2 x 1.2513 x s

∴ s = 159.8366 m

Using kinematical equation:
v = u + at

∴ 0= 20 – 1.2513t

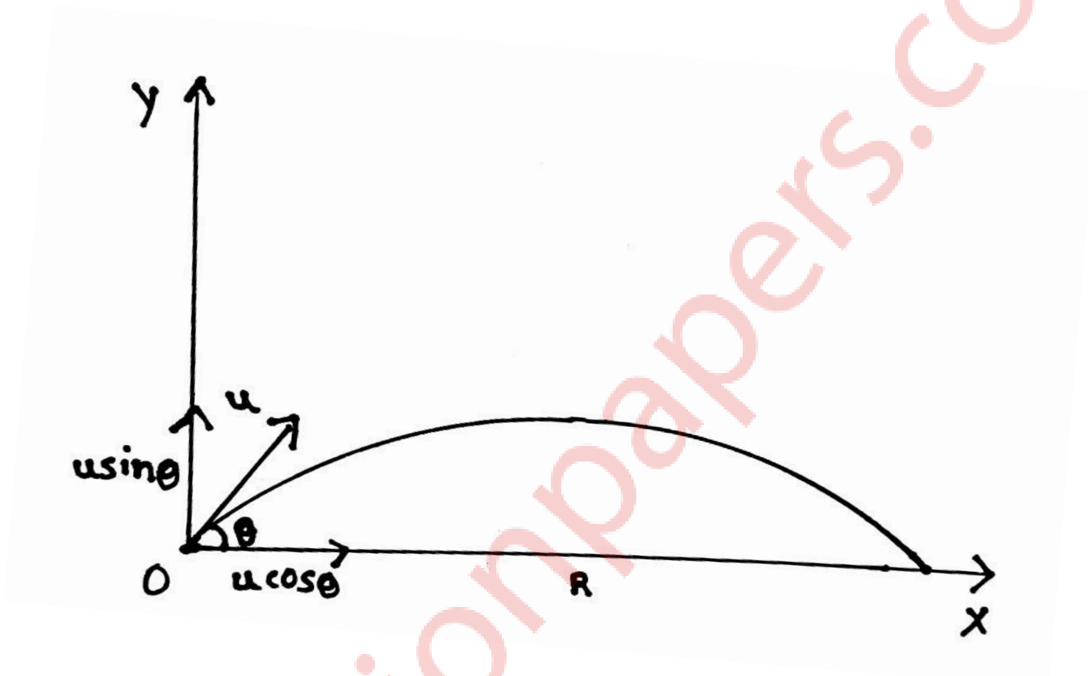
∴ t = 15.9837 s

∴ Distance travelled by the block before stopping = 159.8366 m

∴ Time taken by the block before stopping = 15.9847 s

Q.6b)Derive the equation of path of a projectile and hence show that equation of path of projectile is a parabolic curve. (5 marks)

Solution :



Let us assume that a projectile is fired with an initial velocity u at an angle θ with the horizontal.
Let t be the time of flight.
Let x be the horizontal displacement and y be the vertical displacement.

HORIZONTAL MOTION :

In the horizontal direction,the projectile moves with a constant velocity.

Horizontal component of initial velocity u is u.cosθ

Displacement = velocity x time

$x = u.\cos\theta \times t$

$t = \frac{x}{u\cos\theta}$

VERTICAL MOTION OF PROJECTILE:

In the vertical motion,the projectile moves under gravity and hence this is an accelerated motion.
Vertical component of initial velocity u = u.sinθ
Using kinematics equation :
$s = u_y t + \frac{1}{2} \times a \times t^2$

$y = u\sin\theta \times \frac{x}{u\cos\theta} - \frac{1}{2} \times g \times (\frac{x}{u\cos\theta})^2$

$$\mathbf{y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}}$$

This is the equation of the projectile

This equation is also the equation of a parabola

Thus, proved that path traced by a projectile is a parabolic curve.

Q.6c)A particle is moving in X-Y plane and it's position is defined by

$\overline{r} = (\frac{3}{2}t^2)\overline{\imath} + (\frac{2}{3}t^3)\overline{\jmath}$ .Find radius of curvature when t=2sec. (5 marks)

Solution :

Given : $\overline{r} = (\frac{3}{2}t^2)\overline{\imath} + (\frac{2}{3}t^3)\overline{\jmath}$

To find : Radius of curvature at t = 2sec.

Solution :

Differentiating $\bar{r}$ w.r.t to t

$\frac{d\bar{r}}{dt} = \bar{v} = (\frac{3}{2} \times 2t)\bar{\imath} + (\frac{2}{3} \times 3t^2)\bar{\jmath}$

$\bar{v} = 3t\bar{\imath} + 2t^2\bar{\jmath}$

Once again differentiating w.r.t to t

$\frac{d\,\bar{v}}{dt} = \bar{a} = 3\bar{\imath} + 4t\bar{\jmath}$

$\bar{a}$ = 3$\bar{\imath}$ +4t$\bar{\jmath}$

At t=2s

$\bar{v}$ = (3 x 2) $\bar{\imath}$ +(2x 2^2) $\bar{\jmath}$

= 6$\bar{\imath}$ + 8$\bar{\jmath}$

$\bar{a}$ = 3$\bar{\imath}$ + (4 x 2)$\bar{\jmath}$

=3$\bar{\imath}$ + 8$\bar{\jmath}$

v = |$\bar{v}$ | = $\sqrt{6^2 + 8^2}$

=10 m/s

$$\bar{a} \times \bar{v} = \begin{vmatrix} i & j & k \\ 3 & 8 & 0 \\ 6 & 8 & 0 \end{vmatrix}$$

= i(0-0) -j(0-0) + k(24-48)

= -24k

|$\bar{a}$ x $\bar{v}$| = 24

Radius of curvature = $\frac{v^3}{|\bar{a} \times \bar{v}|} = \frac{10^3}{24}$

=41.6667 m

Radius of curvature at t=2 s is 41.6667 m

Q.6 d) A Force of 100 N acts at a point P(-2,3,5)m has its line of action passing through Q(10,3,4)m. Calculate moment of this force about origin (0,0,0). (5 marks)

Solution :

Given: O = (0,0,0)

P= (4.5, -2)

Q= (-3,1,6)

A= (3,2,0)

F=100 N

To find : Moment of the force about origin

Solution:

Let $\bar{p}$ and $\bar{q}$ be the position vectors of points P and Q with respect to the origin O

∴ $\overline{OP}$ = -2$\bar{\imath}$ +3$\bar{\jmath}$ + 5$\bar{k}$

∴$\overline{OQ}$ = 10$\bar{\imath}$ +3$\bar{\jmath}$ + 4$\bar{k}$

∴ $\overline{PQ}$ = $\overline{OQ}$ -$\overline{OP}$ = (10$\bar{\imath}$ +3$\bar{\jmath}$ +4$\bar{k}$) – (-2$\bar{\imath}$ +3$\bar{\jmath}$ +5$\bar{k}$)

= 12$\bar{\imath}$ – $\bar{k}$

∴ | PQ | = $\sqrt{12^2 + (-1)^2} = \sqrt{145}$

Unit vector along PQ = $\widehat{PQ} = \frac{\overline{PQ}}{|PQ|} = \frac{12\bar{\imath} - \bar{k}}{\sqrt{145}}$

Force along PQ = $\bar{F}$ = 100 x $\frac{12\bar{\imath} - \bar{k}}{\sqrt{145}}$

Moment of F about O = $\overline{OP}$ x $\bar{F}$

$$= \frac{100}{\sqrt{145}} \text{ x } \begin{matrix} \bar{\imath} & \bar{\jmath} & \bar{k} \\ -2 & 3 & 5 \\ 12 & 0 & -1 \end{matrix}$$

= 8.3045 (-3$\bar{\imath}$ + 58$\bar{\jmath}$ − 36– $\bar{k}$)

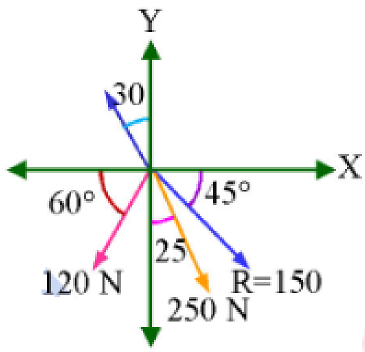
= -24.9135$\bar{\imath}$ + 481.661$\bar{\jmath}$ – 298.962$\bar{k}$ Nm

Moment of the force = -24.9135$\bar{\imath}$ + 481.661$\bar{\jmath}$ – 298.962$\bar{k}$ Nm

ENGINEERING MECHANICS – SEMESTER 1

CBCGS MAY 18

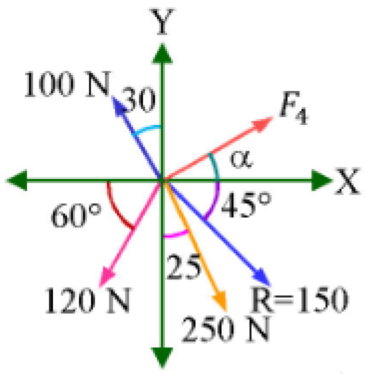
Q1] a) Find fourth force(F_4) completely so as to give the resultant of the system force as shown in figure. (4)



Solution:-

Let F_4 act as an angle α as shown in the figure.

Given, resultant of forces F_1, F_2, F_3 and F_4 is R = 150N



Resolving the forces along Y-axis ,

100cos30-120sin60-250cos25+F_4sinα = -150sin45

$F_4\sin\alpha = -150sin45 - 100cos30 + 120sin60 + 250cos25$

$F_4\sin\alpha = 137.8314$ ………………(1)

Resolving the forces along X-axis,

-100sin30-120cos60+250sin25+F_4cosα = 150cos45

$F_4\cos\alpha = 110.4115$N ………………(2)

Squaring and adding (1) and (2),

$(F_4 \sin\alpha)^2 + (F_4 \cos\alpha)^2 = (137.8314)^2 + (110.4115)^2$

$F_4{}^2(sin^2\alpha + cos^2\alpha) = 18997.5052 + 12190.6887$

$F_4{}^2 = 31188.1939$

$F_4 = 176.6018\text{N}$

Dividing (1) by (2), $\frac{F_4 \sin\alpha}{F_4 \cos\alpha} = \frac{137.8314}{110.4115}$

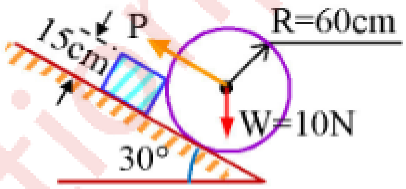
$\tan\alpha = 1.2483$

$\alpha = 51.3031°$

Hence,

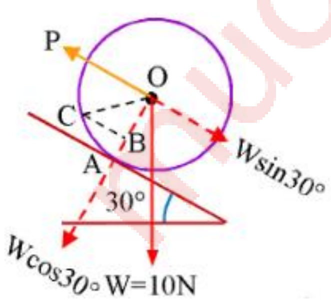
$F_4 = 176.6018\text{N}$, $\alpha = 51.3031°$

Q1] b) Determine the magnitude and direction of the smallest force P required to start the wheel W= 10N over the block. (4)



Solution:-

The simplified figure is as shown



Let point C is the tip of rectangle block from figure

OC = OA = 60cm ....(1)

AB = 15 cm ....(height of the block)

Hence OB = 60-15 = 45cm ....(2)

By Pythagoras theorem

BC = $\sqrt{OC^2 - OB^2} = \sqrt{60^2 - 45^2} = 39.6863$cm

BC = 39.6863cm

When the wheel is about to start. Normal reaction at the point A is zero and

Σ M_C = 0.

$\therefore P \times OB - W cos30 \times BC - W sin30 \times OB = 0$

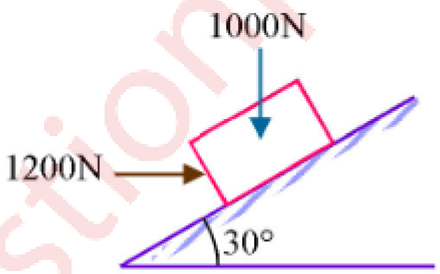
45P = $10cos30 \times 39.6863 + 10sin30 \times 45$

45P = 568.6932.

P = 12.6376N

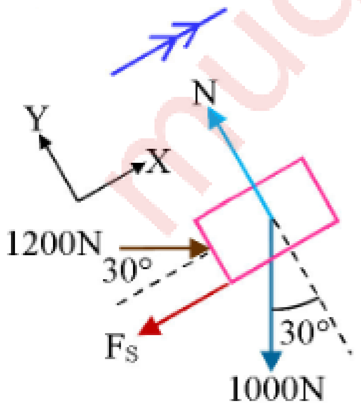
Hence the force P required to start the wheel is 12.6376N.

Q1] c) If a horizontal force of 1200N is applied to the block of 1000N then block will be held in equilibrium or slide down or move up? μ = 0.3. (4)



Solution:-

Let N be normal reaction and F_s be the frictional force



At the instant of impending motion Σ F_Y = 0

Therefore N- 1000cos30 – 1200sin30 = 0

N = 1000cos30 + 1200sin30.

N = 1466.0254N

F_s = μ x N = 0.3(1466.0254)

F_s = 439.8076N. ....(1)

Neglecting friction, net upward up the plane

-1000sin30 + 1200cos30 = 539.2305N. ....(2)

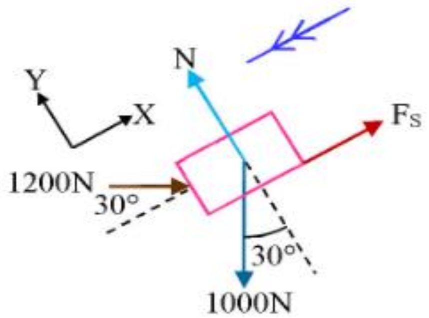
CASE 1:- Block is impending to move up the plane

Σ F_X = -F_s – 1000sin30 + 1200cos30

= -439.8076 + 539.2305. ....(From 1 & 2)

Σ F_X = 99.4229N

Therefore a net force of 99.4229N acts up the plane so the block moves up the plane.



CASE 2:- Block is impending to move down the plane

Σ F_X = F_s – 1000sin30 + 1200cos30

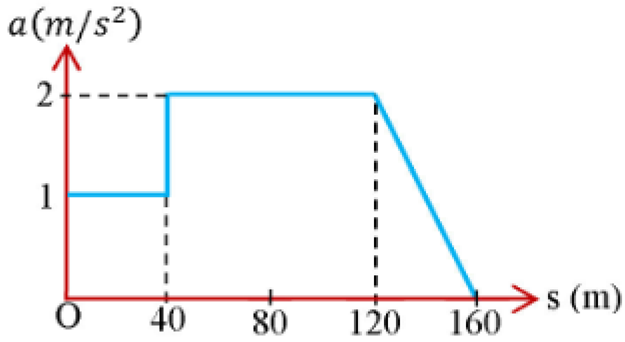
= 439.8076 - 539.2305. ....(From 1 & 2)

Σ F_X = 979.0381N

Therefore a net force of 979.0381N acts up the plane so the block moves up the plane.

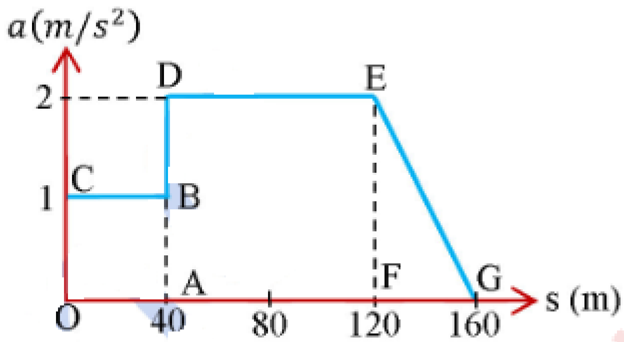
Q1] d) Starting from rest at S = 0 a car travels in a straight line with an acceleration as shown by they a-s graph. Determine the car's speed when S = 20m ,S = 100m,

S =150m. (4)



Solution:-

Part 1:-



For first 40m of journey

a = 1m/s^2

V$\frac{dv}{ds}$ = 1. ....(a = V$\frac{dv}{ds}$)

Vdv = ds

On integration, $\frac{V^2}{2}$= s + c_1 ....(1)

When s = 0, v= 0

c_1= 0 ....(2)

From (1) and (2) $\frac{V^2}{2}$ = s

Therefore V^2 = 2s

When s = 20m, V^2 = 40

Therefore v = 6.3246 m/s

When s = 40m, V^2 = 80

Therefore v = 8.9443 m/s ....(3)

Part 2:-

Motion of car from 40m to 120m

a = 2 m/s^2

V$\frac{dv}{ds}$ = 2.

Vdv = ds

On integration , $\frac{V^2}{2}$ = s +c_1.

V^2 = 4s +2c_1. ....(4)

When s = 40 , = 80 ....from (3)

80 = 160 + 2c_2

-80 = 2c_2. ....(5)

From (4) and (5)

V^2= 4s – 80

When s = 100m , V^2 = 400-80 = 320

v = 17.8885 m/s

When s = 120m , = 480-80 = 400

V = 20m/s. ....(6)

Part 3:-

Motion of car from 120m to 160m

E(120,2) and F(160,0)

Using two-point from equation of EF is

$\frac{a-2}{2-0} = \frac{s-120}{120-160}$.

-20a + 40 = s- 120

160-s =20a

160-s = 20 V

(160-s)ds = 20vdv

On integration, 160s – = 20 + C_3 ....(7)

When s = 120, v = 20m/s. ....from (6)

160 x 120 – (0.5) x 120 x 120 = 10 x 20 x20 + C_3

C_3 = 8000. ....(8)

From (7) and (8)

$$160s - \frac{s^2}{2} = 20 \times \frac{v^2}{2} + c^3$$

When s = 150m, 160(150) $-\frac{1}{2}s^2$ = $10v^2$ -8000

v^2= 475

V = 21.7945m/s

Hence when s = 20m , v = 6.3246m/s

When s = 100m, v = 17.8885m/s

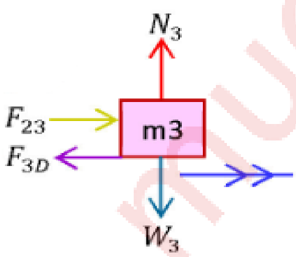
When s = 150m, v = 21.7945m/s.

Q1] e) Three m1, m2, m3 of masses 1.5kg, 2kg and1kg respectively are placed on a rough surface with coefficient of friction 0.20 as shown. If a force F is applied to accelerate the blocks at 3m/s^2 . What will be the force that 1.5kg block exerts on 2kg block? (4)

Solution:-

m_1= 1.5kg, m_2= 2kg, m_3 = 1kg , μ=0.20 , a= 3m/s^2

For m3:-



Weight w_3 = m_3 x g = 1g

Normal reaction N_3 = W_3 = g

Dynamic friction force= F_3 = μN_3 = 0.2g

Let the force exerted by m2 on m3 be F23

By Newton 2nd law

ΣF = ma

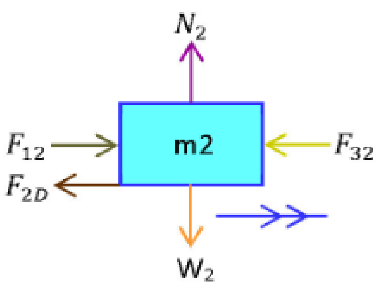
F_{23} - F_3 = m_3 x a

F_{23} =F_3 D + m_3 x a

F_{23} = 0.2g + 1a ....(1)

Similarly,

For m_2:-



Weight W_2= m_2g

Normal reaction N_2 = W_2 = 2g

Dynamic friction force =F_2 D = μ N_2= 0.2 x 2g = 0.4g.

Let the force exerted by m_1 on $m_2$be F_{12}

By Newton's 2nd law

ΣF = ma

F_{12} - F_{32} - F_2 = m_2x a

F_{12} = F_{32} + F_2 + = m_2 x a

= (0.2g + 1a) + 0.4g + 2a. ....from (1)

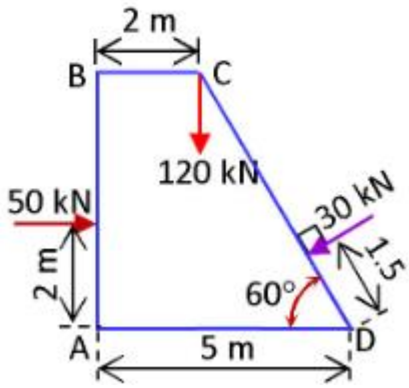
= 0.6g + 3a

= 0.6 x 9.81 + 3 x 3

= 14.886 N

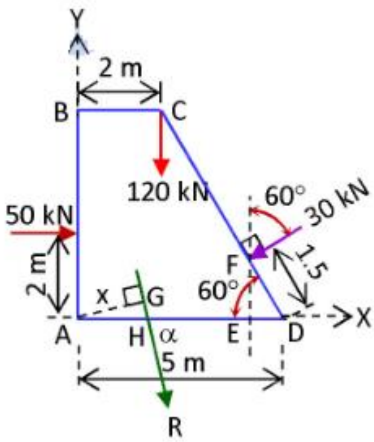
The force that 1.5 kg block exerts on 2kg block = 14.886N.

Q2] a) A dam is subjected to three forces as shown in fig. determine the single equivalent force and locate its point of intersection with base AD (6)



Solution:-

Let R be the resultant and let it act at an angle α to the horizontal



In ΔFED , FD = 1.5cm

∴ FE = FDsin60° = 1.2990 and ED = FDcos60° = 0.75

∴ AE = AD-AE = 5-0.75 = 4.25

Resolving the forces along X-axis, $R_X = 50 - 30sin60$ =24.0192N

Resolving the forces along Y-axis, $R_Y = -120 - 30cos60$ = -135

$\therefore R = \sqrt{{R_X}^2 + {R_Y}^2} = \sqrt{(24.0192)^2 + (-135)^2}$ = 137.1201N

And , $\alpha = \tan^{-1}\left(\frac{R_Y}{R_X}\right) = \tan^{-1}\left(\frac{-135}{24.0192}\right) = -79.9115°$

Resultant moment at A = -50$\times$ $2 - 120 \times 2 - 30sin60° \times 4.25 + 30sin60° \times 1.2990$

A = -370N m

By Varignon's Theorem,

$\therefore 370 = 137.1201 \times X$

X = 2.6984m

In ΔAGH, $\sin\alpha = \frac{x}{AH}$

AH = $\frac{x}{\sin\alpha} = \frac{2.6984}{\sin(79.9115)} = 2.7407$m

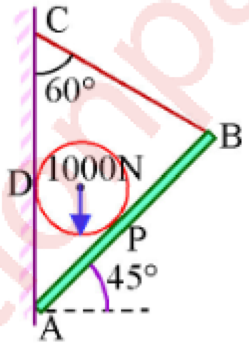
Hence,

Resultant force = 137.1201N (79.9115°)

Resultant moment = 370N m

Resultant cuts base AD at a distance of 2.7407m right of A

Q2] b) A cylinder weighing , 1000N and 1.5m diameter is supported by a beam AB of length 6m and weight 400N as shown. Neglecting friction at the surface of contact of the cylinder. Determine (1) wall reaction at 'D'; (2) hinged reaction at support 'A'; (3) tension in the cable BC (8)

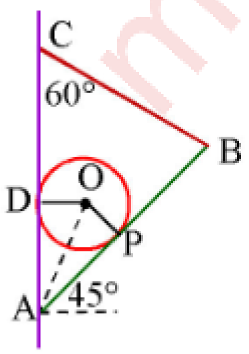


Solution:-

Diameter of cylinder = 1.5m

Radius OD = OP =0.75m

Also, AB =6



Now, ∠DAP = 90° $- 45° = 45°$

Also, $\Delta AOP \cong \Delta AOD$

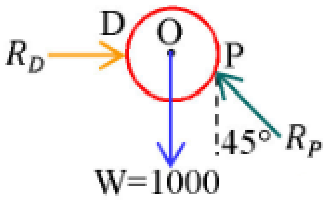
∠OAP = $\frac{1}{2} \times 45 = 22.5°$

In ΔAOP, ∠OPA =90°

tan∠OAP = $\frac{OP}{AP}$

tan22.5 = $\frac{0.75}{AP}$

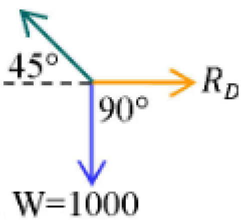
AP = = $\frac{0.75}{tan22.5} = 1.8107$m



FBD of cylinder is as shown

Since the cylinder is in equilibrium,

By Lami's theorem.



$\frac{W}{\sin(180-45)} = \frac{R_D}{\sin(90+45)} = \frac{R_P}{sin90}$

$\frac{1000}{sin135} = \frac{R_D}{sin135}$ and $\frac{1000}{sin135} = \frac{R_P}{1}$

$R_D = 1000$N and $R_P = 1414.2136$N

FBD of beam AB is as shown

Let Q be mid-point of AB

AQ= 3m

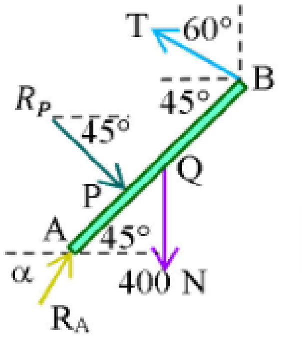
Since beam AB is in equilibrium,

$\sum M_A = 0$

$-R_P \times AP - 400 \times AQcos45 + Tsin(30 + 45) \times AB = 0$

-1414.2136$\times 1.8107 - 400 \times 3 \times 0.7071 + T \times 0.9659 \times 6 = 0$

T = 588.266N



Also, $\sum F_X = 0$

$R_A cos\alpha + R_P cos45 - Tsin60 = 0$

$R_A cos\alpha + 1414.2136 \times 0.7071 - 588.266 \times 0.866 = 0$

$R_A cos\alpha = -490.5676$ …………………(1)

And $\sum F_Y = 0$

$R_A sin\alpha - 400 - R_P sin45 + Tcos60 = 0$

$R_A sin\alpha - 400 - 1414.2136 \times 0.7071 + 588.266 \times 0.5 = 0$

$R_A sin\alpha = 1105.8790$ ……………………(2)

Squaring and adding (1) and (2)

${R_A}^2 cos^2\alpha + {R_A}^2 sin^2\alpha = (-490.5676)^2 + (1105.8790)^2$

$R_A = 1209.8037$N

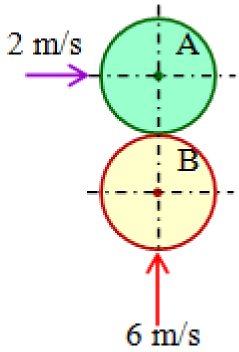
Dividing, (2) by (1) , $\frac{R_A sin\alpha}{R_A cos\alpha} = \frac{1105.8790}{490.5676}$

$\alpha = \tan^{-1} 2.25431 = 66.0779°$

Hence,

1. Wall reaction at D = R_D= 1000N
2. Hinged reaction at support A = 1209.8037

Q2] c) Two balls of 0.12kg collide when they are moving with velocities 2m/sec and 6m/sec perpendicular to each other as shown in fig. if the coefficient of restitution between 'A' and 'B' is 0.8 determine the velocity of 'A' and 'B' after impact (6)



Solution:-

$m_1 = m_2 = 0.12kg$ $u_{1x} = 2m/s$ $u_{1y} = 0m/s$ $u_{2x} = 0m/s$ $u_{2y} = 6m/s$ c =0.8

Case1 :- line of impact is X-axis

Velocities along Y-axis remains constant

$v_{1y} = u_{1y} = 0m/s$ and $v_{2y} = u_{2y} = 6m/s$ ………….(1)

By law of conservation of momentum,

$m_1u_{1x} + m_2u_{2x} = m_1v_{1x} + m_2v_{2x}$

$0.12\times 2 + 0.12 \times 0 = 0.12 \times v_{1x} + 0.12 \times v_{2x}$

Dividing by 0.12 , 2 = $v_{2x} + v_{1x}$ ……………(2)

Also, coefficient of restitution = e = $\frac{v_{2x} - v_{1x}}{u_{1x} - u_{2x}}$

0.8 = $\frac{v_{2x} - v_{1x}}{2-0}$

1.6 = $v_{2x} - v_{1x}$ ……………(3)

Solving (2) and (3)

$v_{2x} = 1.8m/s$ and $v_{1x} = 0.2m/s$ ………………..(4)

From (1) and (4)

$v_1 = \sqrt{{v_{1x}}^2 + {v_{1y}}^2}$ and $v_2 = \sqrt{{v_{2x}}^2 + {v_{2y}}^2}$

$v_1 = \sqrt{0.2^2 + 0^2}$ and $v_2 = \sqrt{1.8^2 + 6^2}$

$v_1 = 0.2m/s$ and $v_2 = 6.2642m/s$

Also after impact

Velocity of ball A = 0.2m/s

Velocity of ball B = 6.2642 m/s

Case 2:- line of impact is y-axis

Velocities along x-axis remains constant

$v_{1x} = u_{1x} = 2m/s$ and $v_{2x} = u_{2x} = 0m/s$ ....(5)

By law of conservation of momentum,

$m_1 u_{1y} + m_2 u_{2y} = m_1 v_{1y} + m_2 v_{2y}$

$0.12\times 0 + 0.12 \times 6 = 0.12 \times v_{1y} + 0.12 \times v_{2y}$

Dividing by 0.12 , 6 = $v_{2y} + v_{1y}$ ....(6)

Also, coefficient of restitution = e = $\frac{v_{2y} - v_{1y}}{u_{1y} - u_{2y}}$

0.8 = $\frac{v_{2y} - v_{1y}}{0-6}$

-4.8 = $v_{2y} - v_{1y}$ ....(7)

Solving (6) and (7)

$v_{2y} = 0.6m/s$ and $v_{1y} = 5.4m/s$ ....(8)

From (4) and (8)

$v_1 = \sqrt{{v_{1x}}^2 + {v_{1y}}^2}$ and $v_2 = \sqrt{{v_{2x}}^2 + {v_{2y}}^2}$

$v_1 = \sqrt{2^2 + 5.4}$ and $v_2 = \sqrt{0^2 + 0.6^2}$

$v_1 = 5.7585m/s$ and $v_2 = 0.6m/s$

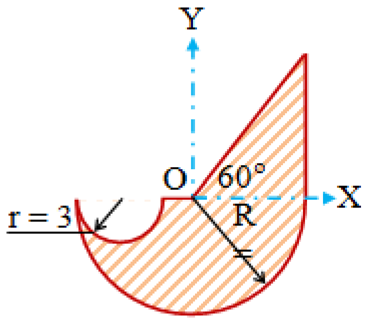
Also, $\alpha_1 = \tan^{-1}\left(\frac{v_{1y}}{v_{1x}}\right)$ = $\tan^{-1}\left(\frac{5.4}{2}\right)$ = 69.67°

Hence, after impact

Velocity of ball A = 5.7585 m/s

Velocity of ball B = 0.6m/s

Q3] a) Find the centroid of the shaded portion of the given area shown in figure (8)



(All Dimensions in cm)

Solution:-

In ΔAOB

OB = 8cm

tan60 = $\frac{AB}{BO}$ $\therefore \sqrt{3} = \frac{AB}{8}$ ∴ AB = $8\sqrt{3}$

Also OD = 8cm and CD = 3cm

OC = 8-3=5cm

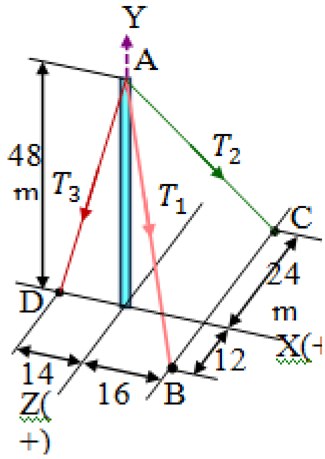
SR NO	PART	Area(in cm^2)	X-co-ord of C.G.(x_1)	Y-co-ord of C.G.(y_1)	Ax_1	Ay_1
1)	Triangle AOB B =8, H= $8\sqrt{3}$	= 0.5BH = 0.5× 8 × $8\sqrt{3}$ =55.4256	$8-\frac{B}{3}=8-\frac{8}{3}$ = 5.3333	$\frac{H}{3}=\frac{8\sqrt{3}}{3}$ =4.6188	295.6033	256.000
2)	Semicircle radius(R)=8	$0.5\pi R^2$ =0.5× $8^2\pi$ =100.5372	0	$-\frac{4R}{3\pi}=\frac{-4\times 8}{3}$ = -3.3955	0.0000	-341.333
3)	Cut semicircle radius(r) = 3	-$0.5\pi r^2$ =-0.5× $3^2\pi$ =-14.1372	-5	$-\frac{4r}{3\pi}=\frac{-4\times 3}{3}$ = -1.2732	70.6858	18.000
	Total	141.8193			366.2891	-67.3333

$$\bar{x} = \frac{\sum Ax}{\sum A} = \frac{366.2891}{141.8193} = 2.5828$$

$$\bar{y} = \frac{\sum Ay}{\sum A} = \frac{-67.3333}{141.8193} = -0.4748$$

Hence centroid is equals to (2.5828,0.4748)

Q3] b) Knowing that the tension in AC is $T_2 = 20kN$ determine required values $T_1$and T_3so that the resultant of the three forces are 'A' is vertical. Also, calculate this resultant. (6)



Solution:-

From figure we observe,

A = (0,48,0) ; B =(16,0,12); C = (16,0,-24) ; D = (-14,0,0);

Let $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$ be the position vectors of points A,B,C and D respectively w.r.t. to origin 0.

$\therefore \overline{OA} = \bar{a} = 48\vec{j}$; $\overline{OB} = \bar{b} = 16\vec{j} + 12\vec{k}$;

$\overline{OC} = \bar{c} = 16\vec{\imath} - 24\vec{k}$; $\overline{OD} = \bar{d} = -14\vec{\imath}$;

Now,

$$\overline{AB} = \bar{b} - \bar{c} = 16\vec{j} + 12\vec{k} - 48\vec{j}$$

$$\overline{AC} = \bar{c} - \bar{a} = 16\vec{\imath} - 24\vec{k} - 48\vec{j}$$

$$\overline{AD} = \bar{d} - \bar{a} = -14\vec{\imath} - 48\vec{j}$$

Magnitude,

$$|\overline{AB}| = \sqrt{(16)^2 + (-48)^2 + (12)^2} = 52$$

$$|\overline{AC}| = \sqrt{(16)^2 + (-48)^2 + (-24)^2} = 56$$

$$|\overline{AD}| = \sqrt{(-14)^2 + (-48)^2} = 50$$

Unit vector,

$$\widehat{AB} = \frac{\overline{AB}}{|\overline{AB}|} = \frac{16\vec{\imath} - 48\vec{j} + 12\vec{k}}{52} = \frac{4}{13}\vec{\imath} - \frac{12}{13}\vec{j} + \frac{3}{13}\vec{k}$$

$$\widehat{AC} = \frac{\overline{AC}}{|\overline{AC}|} = \frac{16\vec{\imath} - 48\vec{j} - 24\vec{k}}{56} = \frac{2}{7}\vec{\imath} - \frac{6}{7}\vec{j} - \frac{3}{7}\vec{k}$$

$$\widehat{AD} = \frac{\overline{AD}}{|\overline{AD}|} = \frac{-14\vec{\imath} - 48\vec{j}}{50} = \frac{-7}{25}\vec{\imath} - \frac{24}{25}\vec{j}$$

Tension in AB = $\overline{T}_1 = T_1\left(\frac{4}{13}\vec{i} - \frac{12}{13}\vec{j} + \frac{3}{13}\vec{k}\right)$

Tension in AC = $\overline{T}_2 = 20\left(\frac{2}{7}\vec{i} - \frac{6}{7}\vec{j} - \frac{3}{7}\vec{k}\right)$

Tension in AD = $\overline{T}_3 = T_3\left(\frac{-7}{25}\vec{i} - \frac{24}{25}\vec{j}\right)$

Net force = $\overline{T}_1 + \overline{T}_2 + \overline{T}_3$ = $T_1\left(\frac{4}{13}\vec{i} - \frac{12}{13}\vec{j} + \frac{3}{13}\vec{k}\right) + 20\left(\frac{2}{7}\vec{i} - \frac{6}{7}\vec{j} - \frac{3}{7}\vec{k}\right) + T_3\left(\frac{-7}{25}\vec{i} - \frac{24}{25}\vec{j}\right)$

= $\left(\frac{4}{13}T_1 + 20 \times \frac{2}{7} - \frac{7}{25}T_3\right)\vec{i} - \left(\frac{12}{13}T_1 + 20 \times \frac{6}{7} + \frac{24}{25}T_3\right)\vec{j} + \left(\frac{3}{13}T_1 - 20 \times \frac{3}{7}\right)\vec{k}$ ....(1)

Given, the resultant at 'A' is vertical i.e., along y-axis

$\frac{3}{13}T_1 - 20 \times \frac{3}{7}$ & $\frac{4}{13}T_1 + 20 \times \frac{2}{7} - \frac{7}{25}\ T_3$ ....(2)

$\frac{3}{13}T_1 = 20 \times \frac{3}{7}$

$T_1 = \frac{260}{7} = 37.1429kN$ ....(3)

From (2) and (3), $\frac{4}{13} \times \frac{260}{7} + 20 \times \frac{2}{7} - \frac{7}{25}\ T_3 = 0$

$\frac{120}{7} = \frac{7}{25}\ T_3$

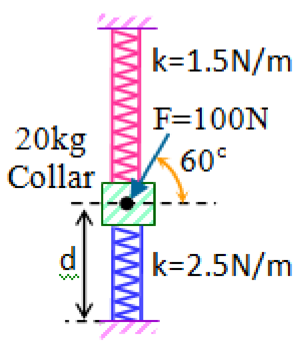
$T_3 = \frac{3000}{49} = 61.2245kN$ ....(4)

From (1), (3) and (4),

Resultant = $-\left(\frac{12}{13} \times \frac{260}{7} + 20 \times \frac{6}{7} + \frac{24}{25} \times \frac{3000}{49}\right)$

Resultant = $-\frac{5400}{49} = -110.2041kN$

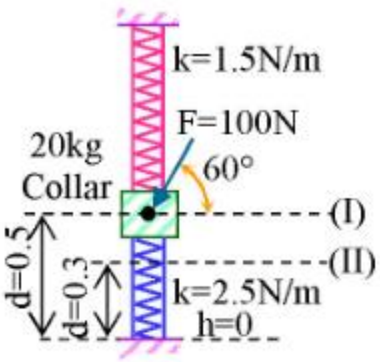
Q3] c) Fig shows a collar of mass 20kg which is supported on the smooth rod. The attached springs are both compressed 0.4m when d =0.5m. determine the speed of the collar after the applied force F=100N causes it to be displaced so that d =0.3m. knowing that collar is at rest when d=0.5m **(6)**



Solution:-

m =20kg,

position 1: when d =0.5m



v =0

$KE_1 = \frac{1}{2}mv_1^2 = 0$ and $PE_1 = mgh = 20g \times 0.5 = 10g$

Compression of Top spring $(x_{T1}) = 0.4$m

Compression of B spring $(x_{B1}) = 0.4$m

Spring energy of top spring $= E_{S_{T1}} = \frac{1}{2}K_{T1}x_{T1}^2 = \frac{1}{2} \times 1.5 \times 0.4^2 = 0.12$J

Spring energy of bottom spring $= E_{S_{B1}} = \frac{1}{2}K_{B1}x_{B1}^2 = \frac{1}{2} \times 2.5 \times 0.4^2 = 0.2$J

Position 2: when d = 0.3m

Let v be the velocity of the block

$KE_2 = \frac{1}{2}mv^2 = \frac{1}{2} \times 20 \times v^2 = 10v^2$ and $PE_2 = mgh = 20g \times 0.3 = 6g$

Compression of top spring (x_{T2})= 0.4-0.2 = 0.2m

Compression of bottom spring (x_{B2})= 0.4+0.2 = 0.6m

Spring energy of top spring = $E_{S_{B2}} = \frac{1}{2}K_{T2}x_{T2}^2 = \frac{1}{2} \times 1.5 \times 0.2^2 = 0.03$J

Spring energy of bottom spring = $E_{S_{B2}} = \frac{1}{2}K_{B2}x_{B2}^2 = \frac{1}{2} \times 2.5 \times 0.6^2 = 0.45$J

Work done by the vertical component of the force = W =100sin60× 0.2 = 17.32015J

Applying work energy principle for the position (1) and (2) , $U_{1-2} = KE_2 - KE_1$

$W + PE_1 - PE_2 + E_{S_{T1}} + E_{S_{B1}} - E_{S_{B2}} - E_{S_{B2}} = KE_2 - KE_1$

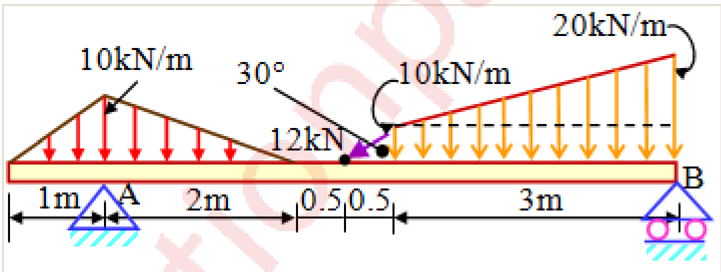
17.3205+10g-6g + 0.12+0.2-0.03-0.45 = $10v^2$

56.3871 = $10v^2$

V = 2.3746 m/s

The speed of the collar after the force F is applied = 2.3746 m/s.

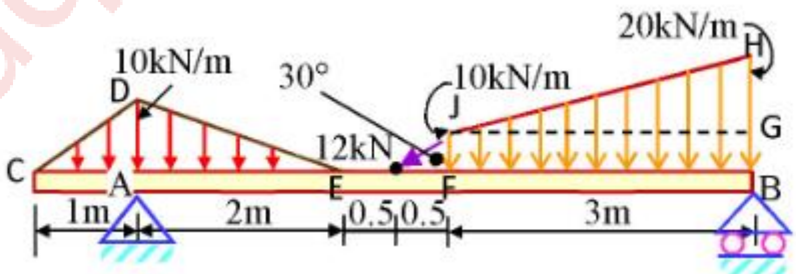
Q4] a) Find the support reactions at point 'A' and 'B' of the given beam (8)



Solution:-

Effective forces of distributed load CAD = $\frac{1}{2} \times 1 \times 10 = 5kN$

It acts as $\frac{1}{3}$m from A



Effective force of distributed load EAD = $\frac{1}{2} \times 2 \times 10 = 10kN$

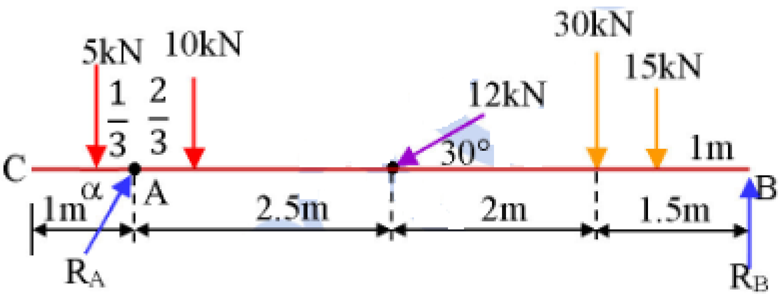
It acts at $\frac{2}{3}$m from A

Effective force of distributed load JFBGJ = $3 \times 10 = 30$kN

If acts at 1.5m from B

Effective force of distributed load JGH = $\frac{1}{2} \times 3 \times (20 - 10) = 15$kN

It acts at 1m from B



Since the beam is in equilibrium $\sum M_A = 0$

$5 \times \frac{1}{3} - 10 \times \frac{2}{3} - 12sin30 \times 2.5 - 30 \times 4.5 - 15 \times 5 + R_B \times 6 = 0$

$-230 + R_B \times 6 = 0$

$R_B = 38.333$kN

Also , $\sum F_X = 0$

$R_A cos\alpha - 12cos30 = 0$

$R_A cos\alpha = 10.3923$kN ………………(1)

And, $\sum F_Y = 0$

$R_A sin\alpha - 5 - 10 - 30 - 12sin30 - 15 + R_B = 0$

$R_A sin\alpha - 66 + 38.333 = 0$

$R_A sin\alpha = 27.6667$kN ……………………(2)

Squaring and adding (1) and (2) ,

$R_A{}^2 cos^2\alpha + R_A{}^2 sin^2\alpha = 10.3923^2 + 27.6667^2$

$R_A = 29.5541$kN

Dividing, (2) by (1), $\frac{R_A sin\alpha}{R_A cos\alpha} = \frac{27.6667}{10.3923}$

$\alpha = \tan^{-1}(2.6622) = 69.4125°$

Hence,

Reaction at A = 29.5541

Reaction at B = 38.3333kN

Q4] b) The motion of the particle is defined by the relation a= 0.8t m/s^2 where 't' is measured in sec. It is found that at X = 5cm,V = 12m/sec when t = 2sec find the position and velocity at t = 6sec. (6)

Solution:-

Given :- a = 0.8t $\frac{dv}{dt} = 0.8t$

dv = 0.8tdt

On integration

V = 0.8 x $\frac{t^2}{2}$ + c

Therefore , V = 0.4t^2+ c. ....(1)

Given ,when t = 2 then V = 12

12 = 0.4 x 2^2 + c

12 = 1.6 + c

c = 10.4 ....(2)

From (1) & (2)

V = 0.4t^2 + 10. ....(3)

Therefore $\frac{dX}{dt}$ = 0.4t^2 + 10.

dX = (0.4t^2 + 10)dt

On integration, X = 0.4 x $\frac{t^3}{3}$ + 10.4t + k ....(4)

Given , when t = 2 then X = 5

5 = 0.4 x $\frac{2^3}{3}$ + 10.4 x 2 + k

k = -253/15. ....(5)

From (4) and (5)

X = 0.4 x $\frac{t^3}{3}$ + 10.4t + k

X = 0.4 x $\frac{t^3}{3}$ + 10.4t – $\frac{253}{15}$

When t = 6,

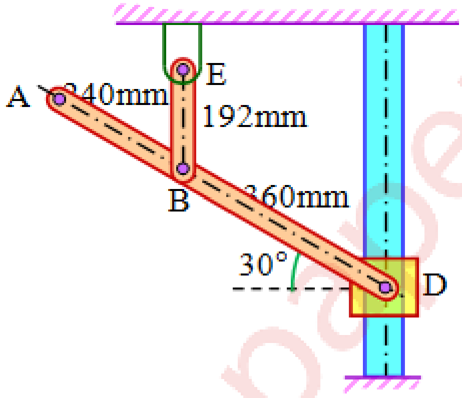
From (3) V = 0.4 x 6 x 6 + 10.4 = 24.8m/s

From (6) X = $0.4 \times \frac{6^3}{3} + 10.4t - \frac{253}{15}$

X = 74.333 from the initial position

Q 4] c) Rod EB in the mechanism shown in the figure has angular velocity of 4 rad/sec at the instant shown in counter clockwise direction. Calculate

1) Angular velocity of AD 2) velocity of collar 'D'. 3) velocity or point 'A' (6)



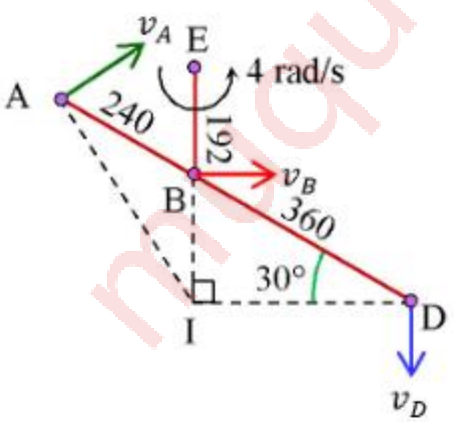
Solution:-

ω_{EB} = 4rad/sec , EB = 192mm = 0.192 m

AB = 240mm =0.24m , DB =360mm = 0.36m

Instantaneous center of rotation is the point of intersection of $\overrightarrow{v_A}$ and $\overrightarrow{v_B}$.

Let I be the ICR as shown in the figure



In Δ BID

∠BDI = 30° , ∠BID = 90°

∠IBD = 180-30° − 90° = 60°

IB =0.36sin30 = 0.18 ....(1) and

ID = 0.36cos30 = 0.3117m ....(2)

Also ∠IBA =180- 60° = 120° ....(3)

In Δ IBA, by cosine rule

$IA^2 = IB^2 + AB^2 - 2IA \times AB \times cos\angle\text{IBA}$

$IA^2 = 0.18^2 + 0.24^2 - 2 \times 0.18 \times 0.24 \times cos120°$ ....(from 1)

IA = 0.3650m ....(4)

By sine rule, $\frac{AB}{sinI} = \frac{IB}{sinA} = \frac{IA}{sinB}$

$\frac{0.24}{sinI} = \frac{0.3650}{sin120}$ ....(from 3 & 4)

$\text{sinI} = \frac{0.24 \times sin120°}{0.3650} = 0.5694$

∠AIB = 34.7113°

Now, instantaneous velocity of point B = rω

$v_B = EB \times \omega_{EB} = 0.192 \times 4 = 0.768 m/s$ ....(5)

Angular velocity of the rod AD = $\omega_{AD} = \frac{v_B}{r} = \frac{v_B}{IB} = \frac{0.768}{0.18} = 4.2667 rad/s$ ....(from 1 & 5) ......(6)

Instantaneous velocity of point D = rω

= ID× $\omega_{AD} = 0.3117 \times 4.2667 = 1.3302 m/s$ ....(from 2 & 6)

And instantaneous velocity of point A = rω

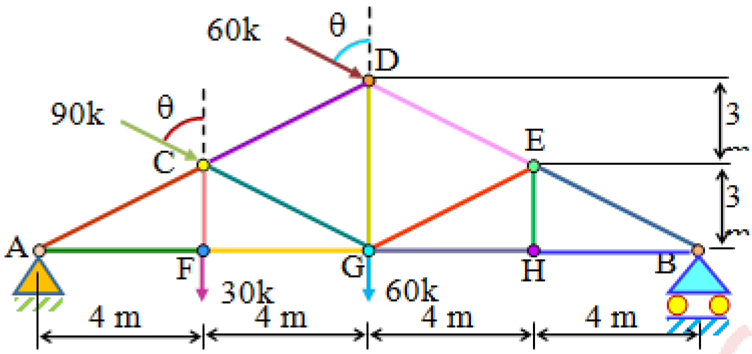
= IA× $\omega_{AD} = 0.3650 \times 4.2667 = 1.5572 m/s$ ....(from 4 & 6)

Hence,

1. Angular velocity of AD = 4.2667 rad/sec
2. Velocity of collar 'D' = 1.3302 m/s

Velocity or point A = 1.5572 m/s

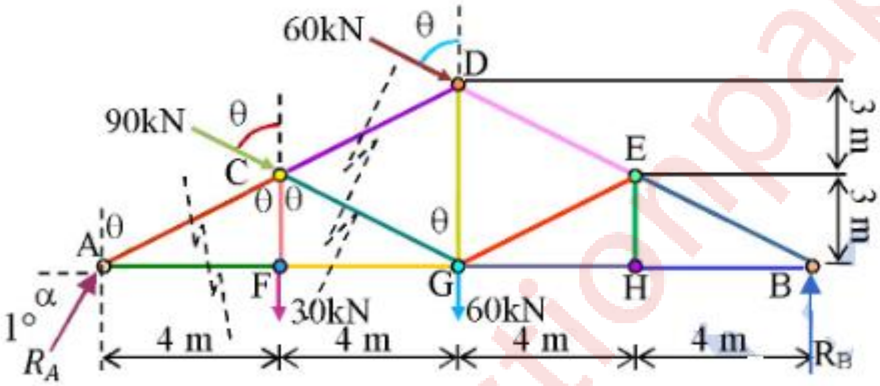
Q5] a) A simply supported pin jointed truss is loaded and supported as shown in fig, (1) identify the members carrying zero forces (2) find support reactions. (3) find forces in members CD, CG, FG and CF using method of section (8)



Solution:-

In ΔGFC, $\tan\theta = \frac{GF}{CF} = \frac{4}{3}$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.1301°$



$\sin\theta = 0.8$ and $\cos\theta = 0.6$ ....(1)

zero force members:

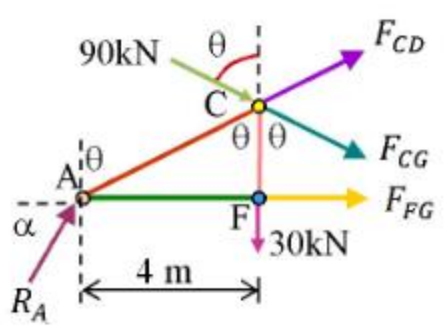
loading at joint H is as shown.

Member EH will have zero force.

Similarly, after EH is removed loading at joint E is as shown

Member EG will have zero force.

Support reactions:



As the truss is in equilibrium, $\sum M_A = 0$

$-30 \times 4 - 90cos\theta \times 3 - 60cos\theta \times 8 - 60sin\theta \times 6 - 60 \times 8 + R_B \times 16 = 0$

-120-360$\times 0.6 - 270 \times 0.8 - 480 \times 0.6 - 360 \times 0.8 - 480 + 16R_B = 0$ ....(from 1)

-1608+16$R_B = 0$

$R_B = 100.5kN$

Also, $\sum F_Y = 0$

$R_A sin\alpha - 30 - 60 - 90cos\theta - 60cos\theta + R_B = 0$

$R_A sin\alpha = 79.5kN$ ....(2)

And, $\sum F_X = 0$

$R_A cos\alpha + 90sin\theta + 60sin\theta = 0$

$R_A cos\alpha + 150 \times 0.8 = 0$

$R_A cos\alpha = -120kN$ ....(3)

Squaring and adding (2) and (3),

$(R_A sin\alpha)^2 + (R_A cos\alpha)^2 = (79.5)^2 + (-120)^2$

$R_A^2(sin^2\alpha + cos^2\alpha) = 6320.25 + 14400$

$R_A^2 = 20720.25$

$R_A = 143.9453kN$

Dividing, (2) and (3) , we get

tan$\alpha = 0.6625$

$\alpha = 33.5245°$

Method of sections:

Applying conditions of equilibrium to the section as shown $\sum M_A = 0$

-30$\times$ 4 $- (90 + F_{CG})cos\theta \times 4 - (90 + F_{CG})sin\theta \times 3 = 0$

-120-(90+F_{CG}) $\times 0.6 \times 4 - (90 + F_{CG}) \times 0.8 \times 3 = 0$ ....(from 1)

-4.8(90+F_{CG}) $= 120$

$F_{CG} = -115kN$ ……………….(4)

Also, $\sum F_Y = 0$

$R_A sin\alpha - 30 - F_{CG}cos\theta + F_{CD}cos\theta - 90cos\theta = 0$

79.5-30-115$\times 0.6 + F_{CD} \times 0.6 - 90 \times 0.6 = 0$

64.5+0.6$F_{CD} = 0$

$F_{CD} = -107.5kN$ ………………….(5)

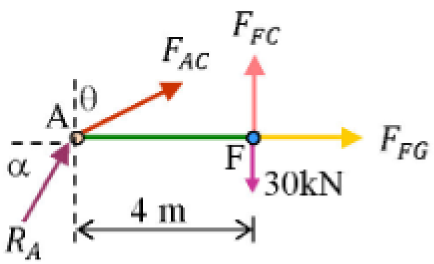
And, $\sum F_X = 0$

$R_A cos\alpha + F_{FG} + F_{CG}sin\theta + F_{CD}sin\theta + 90sin\theta = 0$

-120+$F_{FG} - 115 \times 0.8 - 107.5 \times 0.8 + 90 \times 0.8 = 0$

$F_{FG} = 226kN$

Applying conditions of equilibrium to the section shown below, $\sum M_A = 0$



$F_{FC} \times 4 - 30 \times 4 = 0$

$F_{FC} = 30kN$ …………………(6)

Also, $\sum F_Y = 0$

$R_A sin\alpha - 30 + F_{AC}cos\theta + F_{FC}$=0

79.5 – 30-$F_{AC} \times 0.6 + 30 = 0$

$F_{AC} = 132.5kN$

Members carrying zero forces are EH and EG

Support reactions:-

$R_A = 143.9453kN$

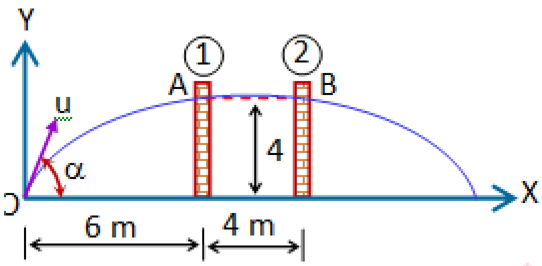
$R_B = 100.5kN$

Forces in members:-

CD = 107.5kN(C) CG = 115kN(C)

FG = 226kN(T) and CF = 30kN(T)

Q5] b) A jet of water discharging from nozzle hits a vertical screen placed at a distance of 6m from the nozzle at a height of 4m. when the screen is shifted by 4m away from the nozzle from its initial position the jet hits the screen again at the same point. Find the angle of projection and velocity of projection of the jet at the nozzle. (6)



Solution:-

The path of the projectile is given by $y = x\tan\theta - \frac{gx^2}{2u^2}sec^2\theta$ ....(1)

The water jet passes through the point A(6,4) and B(10,4)

Substituting , x =6 and y = 4 in (1) we get $4 = 6\tan\theta - \frac{36g}{2u^2}sec^2\theta$ ....(2)

Substituting , x =10 and y = 4 in (1) we get $4 = 10\tan\theta - \frac{100g}{2u^2}sec^2\theta$ ....(3)

Multiplying equation (2) by 25 and equation (3) by 9 and then subtract

$\therefore 100 - 36 = \left(150tan\theta - \frac{900g}{2u^2}sec^2\theta\right) - \left(90tan\theta - \frac{900g}{2u^2}sec^2\theta\right)$

$\therefore 64 = 60tan\theta$

$\theta = \tan^{-1}\left(\frac{64}{60}\right) = 46.8476°$ ....(4)

From (3) and (4) ,

$4 = 10\tan(46.8476) - \frac{100g}{2u^2}sec^2(46.8476)$

$\frac{1048.58}{u^2} = 6.6667$

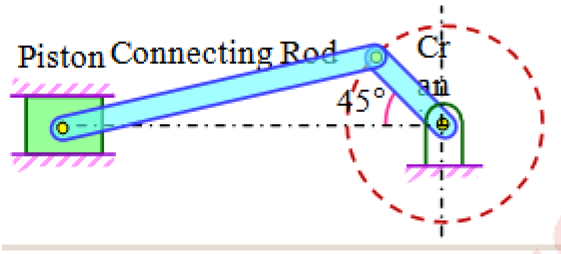
$u^2 = 157.2862$

U = 12.5414 m/s

Hence the angle of projection = 46.8476°

Velocity of projection = 12.5414 m/s

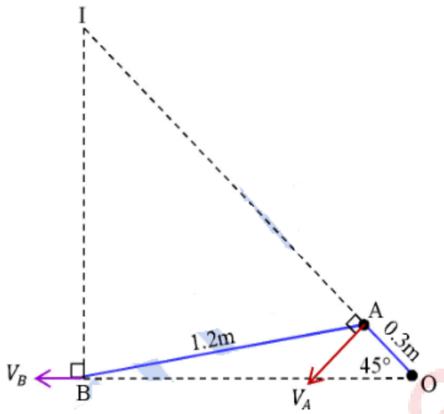
Q5] c) In a crack and connecting rod mechanism the length of crack and connecting rod are 300mm and 1200mm respectively. The crack is rotating at 180 rpm. Find the velocity of piston, when the crack is at an angle of 45° with the horizontal (6)



Solution:-

Let OA and AB be the crank and the connecting rod.

Frequency(n) = 180rpm = $\frac{180}{60}$ = 3rps



OA = 300nm = 0.3m , AB = 1200mm = 1.2m

We assume crank is rotating in anti-clockwise direction

∴ Angular velocity of the crank = $\omega_{AB} = 2\pi n = 2\pi \times 3 = 18.8496$ rad/s

∴ instantaneous velocity of point A = rω

$v_A = OA \times \omega_{OA} = 0.3 \times 18.8496 = 5.6549$ m/s

Instantaneous centre of rotation is the point of intersection of $\overrightarrow{v_A}$ and $\overrightarrow{v_B}$

Let I be the ICR of the connecting rod AB as shown in figure.

In ΔOAB by sine rule, $\frac{AB}{sinO} = \frac{OA}{sinB}$

∴ $\frac{1.2}{sin45} = \frac{0.3}{sin\angle ABO}$

$\therefore sin\angle ABO = \frac{0.3sin45}{1.2} = 0.1768$

$\therefore \angle ABO = 10.1821°$

In Δ IOB , $\angle BIO = 180° - 90° - 45° = 45°$

In Δ IAB, $\angle ABI = 90 - 10.1821° = 79.8179°$

$\therefore \angle IAB = 180° - 45° - 79.8179° = 55.1821°$

In Δ IAB by sine rule, $\frac{AB}{sinI} = \frac{IB}{sinA} = \frac{IA}{sinB}$

$\therefore \frac{1.2}{sin45} = \frac{IB}{sin55.1821} = \frac{IA}{sin79.8179}$

$\therefore \frac{1.2}{sin45} = \frac{IB}{sin55.1821}$ and $\therefore \frac{1.2}{sin45} = \frac{IA}{sin79.8179}$

IB =$\frac{1.2sin55.1821}{sin45} = 1.3931$ and

IA = $\frac{1.2sin79.8179}{sin45} = 1.6703$

Angular velocity of the rod AB = $\omega_{AB} = \frac{v_A}{r} = \frac{v_A}{IA} = \frac{5.6549}{1.6703} = 3.3855$ rad/s

Instantaneous velocity of B = $r\omega_{AB} = IB \times \omega_{AB} = 1.3932 \times 3.3855 = 4.7168$ m/s

Hence, velocity of piston = 4.7168 m/s

Q6] a) Force F = 80i+50j-60k passes through a point A(6,2,6). Compute its moment about a point B(8,1,4) (4)

Solution:-

$\bar{F} = 80\vec{\imath} + 50\vec{\jmath} - 60\vec{k}$

Let $\bar{a}$ and $\bar{b}$ be the position vectors of point A and B respectively.

$\bar{a} = 6\vec{\imath} + 2\vec{\jmath} + 6\vec{k}$ and $\bar{b} = 8\vec{\imath} + 1\vec{\jmath} + 4\vec{k}$

$\overline{BA} = \bar{a} - \bar{b}$ = $(6\vec{\imath} + 2\vec{\jmath} + 6\vec{k})$-$(8\vec{\imath} + 1\vec{\jmath} + 4\vec{k})$ = $-2\vec{\imath} + 1\vec{\jmath} + 2\vec{k}$

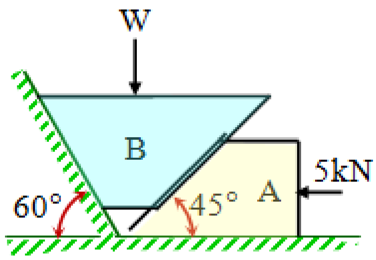
Moment of F about B = $\overline{BA} \times \bar{F}$

B = $\begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ -2 & 1 & 2 \\ 80 & 50 & -60 \end{vmatrix}$ = $\vec{\imath}(-60 - 100) - \vec{\jmath}(120 - 160) + \vec{k}(-100 - 80)$ = $-160\vec{\imath} + 40\vec{\jmath} - 180\vec{k}$

Hence , moment of F about B is -160$\vec{i} + 40\vec{j} - 180\vec{k}$ units

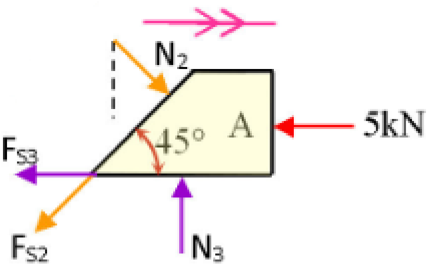
---

Q6] b) A force of 5KN is acting on the wedge as shown in fig. the coefficient of friction at all rubbing surfaces is 0.25. find the load 'W' which can be held in position. The weight of block 'B' may be neglected. (8)



Solution:-

Let N_1, N_2, N_3,be the normal reaction at the surface of contact



$\therefore\ F_{S1} = \mu_1 N_1 = 0.25N_1$, $F_{S2} = \mu_2 N_2 = 0.25N_2$, $F_{S3} = \mu_3 N_3 = 0.25N_3$ ....(1)

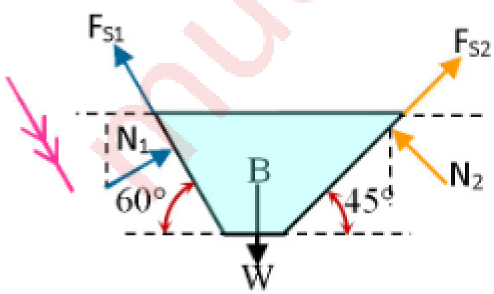
Block A is impending to move towards right.

Since the block A is under equilibrium , $\sum F_Y = 0$

$\therefore N_3 - F_{S2}sin45 - N_2cos45 = 0$

$\therefore N_3 - 0.25N_2 \times 0.7071 - N_2 \times 7071 = 0$ ....(from 1)

$\therefore N_3 - 0.8839N_2 = 0$ ....(2)



Also $\sum F_X = 0$

$-5 - F_{S3} - F_{S2}cos45 + N_2sin45 = 0$

$\therefore -5 - 0.25N_3 - 0.25N_2 \times 0.7071 + N_2 \times 0.7071 = 0$ ………………(from 1)

$\therefore -0.25N_3 + 0.5303N_2 = 5$ …………………(3)

Solving (2) and (3) simultaneously, we get $N_3 = 14.2876$kN and $N_2 = 16.1642$kN ………………(4)

Block B is impending to move down

Since the block B is under equilibrium , $\sum F_X = 0$

$\therefore N_1 sin60 - F_{S1}cos60 + F_{S2}cos45 - N_2 sin45 = 0$

$\therefore 0.866N_1 - 0.25N_1 \times 0.5 + 0.25N_2 \times 0.7071 - N_2 \times 0.7071 = 0$ …………..(from 1)

$\therefore 0.866N_1 - 0.125N_1 + 0.1768 \times 16.1642 - 16.1642 \times 0.7071 = 0$ ……………………(from 4)

$\therefore 0.741N_1 - 8.5719 = 0$

$N_1 = 11.4939$ kN …………………..(5)

Also $\sum F_Y = 0$

$\therefore -W + N_1 cos60 + F_{S1}sin60 + F_{S2}sin45 + N_2 cos45 = 0$

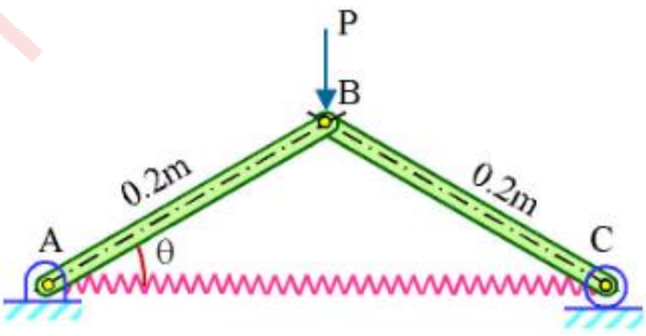
$\therefore N_1 \times 0.5 + 0.25N_1 \times 0.866 + 0.25N_2 \times 0.7071 + N_2 \times 0.7071 = W$ ……………..(from 1)

$\therefore 11.4939 \times 0.5 + 0.2165 \times 11.4939 + 0.1768 \times 16.1642 + 16.1642 \times 0.7071 = W$…..(from 4 and 5)

$\therefore$ W = 22.5225kN

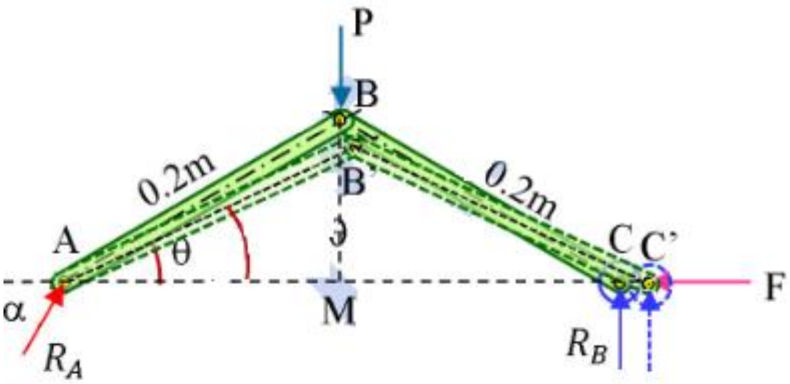
Hence a load of 22.5225kN can be held in the position.

Q6] c) The stiffness of the spring is 600 N/m. find the force 'P' required to maintain equilibrium such that $\theta = 30°$. The spring is unstretched when $\theta = 60°$.neglect weight of the rods. Use method of virtual work. (4)



Solution:-

Principle of virtual work:-



If a body is in equilibrium the total virtual work of forces acting on the body is zero for any virtual displacement.

K = 600N/m, $\theta = 30°$

When $\theta = 60°$

AM = ABcos60° $= 0.2cos60° = 0.1$ and BM = ABsin60° $= 0.2sin60° = 0.1732$m

Given, the spring is unstretched

Unstretched length of the spring = AC = 2AM =0.2m

When $\theta = 30°$

AM = ABcos30° $= 0.2cos30° = 0.1732$ and BM =ABsin30° $= 0.2sin30° = 0.1$m

AC = 2AM =0.4cos30° $= 0.3464$m

Extension in the spring(x) = 0.3464-0.2 = 0.1464m

Spring force(F) = Kx = 600$\times$ 0.1464 = 87.8461N

Let rod AB have a small virtual angular displacement $\delta\theta$ in the clockwise direction

The new position of rods AB' & B'C' is shown dotted

The reaction forces R_A and R_B are not active forces, so they do not perform any virtual work

Let A be the origin and dotted line through A be the X-axis of the system

Consider,

Active forces	Co-ordinate of the point of action along the forces	Virtual displacement
F = 87.8461	X Co-ordinate of C = x_c=0.4cosθ	δx_c= -0.4sinθ $\delta\theta$
P	Y Co-ordinate of C = y_B=0.2sinθ	δy_B= 0.2cosθ $\delta\theta$

By principle of virtual work, $\delta U = 0$

-F$\times$ $\delta x_c - P \times \delta y_B$= 0

-87.8461$\times$ $-0.4sin\theta\delta\theta - P \times 0.2cos\theta\delta\theta = 0$

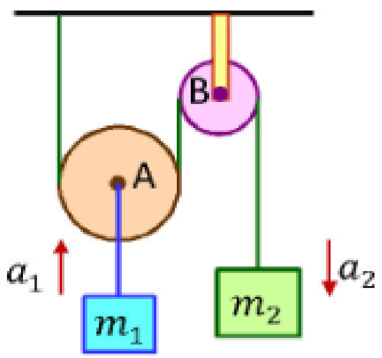
Dividing by $\delta\theta$ and put θ =30, 35.1384sin30° $- 0.2Pcos30° = 0$

$\frac{35.1384sin30°}{0.2\times cos30°} = P$

P= 101.4359N

Hence, the force P required to maintain equilibrium = 101.4359kN

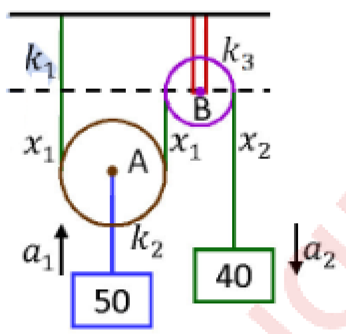
Q6] d) Two masses are interconnected with the pulley system. Neglecting frictional effect of pulleys and cord, determine the acceleration of masses m_1, take m_1 = 50kg and m_2= 40kg. **(4)**



Solution:-

$\boldsymbol{m_1}$ = 50kg and $\boldsymbol{m_2}$ = 40kg

Let x_1 and x_2 be displacement of pulleys A and B respectively.



The string around the pulley is of constant length.

k_1 + x_1 + k_2 + x_1 + k_3 + x_2= L

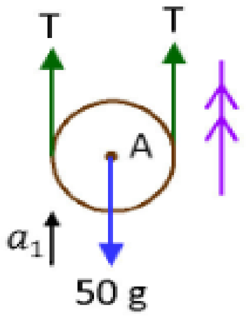
k_1 + k_2+ k_3 + 2x_1 = L - x_2

On differentiating w.r.t 't' we get 2v_1 = -v_2

Where $\frac{dx}{dt}$ = v the velocities of the two pulleys again differentiating w.r.t we get

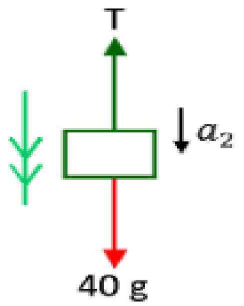
2a_1 = a_2

$a_1 = \frac{1}{2} a_2$ ....(1)



Let T be the Tension in the cord

Applying Newton's 2nd law , $\Sigma F_Y = m_1 a_1$



2T- 50g = 50 x (0.5) a_2 ....from (1)

2T = 50g + 25a_2.

T = 25g +12.5a_2. ....(2) (dividing by 2)

Applying Newton's 2nd law to 40kg block

$\Sigma F_Y = m_2 a_2$

40g-T = 40a_2

40g – (25g +12.5a_2) = 40a_2 ....from (2)

40g – 25g – 12.5a_2 = 40a_2

15g = 52.5a_2

$a_2 = \frac{15g}{52.5}$ = 2.8029 m/s^2

Acceleration of the mass m_2 = 2.8029 m/s^2

MUMBAI UNIVERSITY

SEMESTER – I

ENGINEEERING MECHANICS

QUESTION PAPER – DEC 2018

Q.1. Solve any four.

a) Find the resultant of the parallel force system shown in Figure 1 and locate the same with respect to point C. (5 marks)

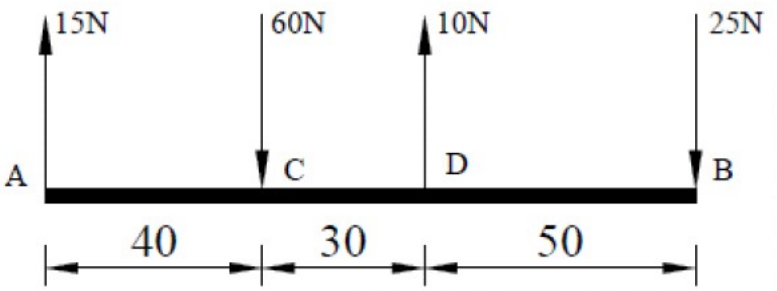


Figure 1

Solution :

$\sum Fx = 0$ → +ve

$\sum Fy = 15 - 60 + 10 - 25$ ↑ +ve

$= -60$ N

$R = \sqrt{Fx^2 + Fy^2}$

$= \sqrt{(-60)^2}$

R = 60 N

$\theta = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$= \tan^{-1}(\frac{-60}{0})$

$= -90^0$

By Varignon's theorem,

$\sum Mc^F = R \times d$ ↻ +ve

d is the distance of the resultant from point C and assume R to be on the right of point C

(15x40) – (10x30) + (25x80) = 60xd

d = 38.333 m

The resultant force is 60 N downwards and is located 38.333 m away to the right of point C.

b) Using Instantaneous Centre of Rotation (ICR) method, find the velocity of point A for the instant shown in Figure 2. Collar B moves along the vertical rod, whereas link AB moves along the plane which is inclined at 25^0. $\Theta = 45^0$ (5 marks)

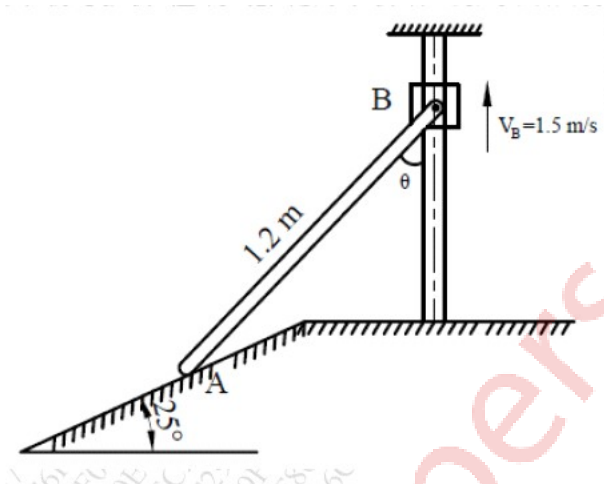
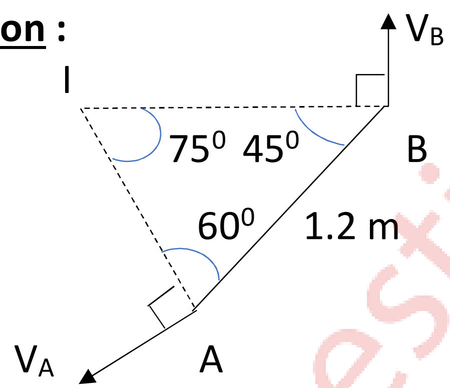


Figure 2

Solution :



By using sine rule,

$$\frac{AB}{\sin I} = \frac{BI}{\sin A} = \frac{AI}{\sin B}$$

$$BI = \frac{1.2\ X \sin(60)}{\sin(75)} = 1.076 \text{ m}$$

$$AI = \frac{1.2\ X \sin(45)}{\sin(75)} = 0.878 \text{ m}$$

ω_{AB} = BI x V_B = 1.076 x 1.5 = 1.614 rad/s

ω_{AB} = AI x V_A

$$V_A = \frac{\omega AB}{AI} = \frac{1.614}{0.878} = 1.838 \text{ m/s}$$

The velocity of point A for the given instance is 1.838 m/s.

c) If the support reaction at A, for the beam shown in Figure 3, is zero, then find force 'P' and the support reaction at B. (5 marks)

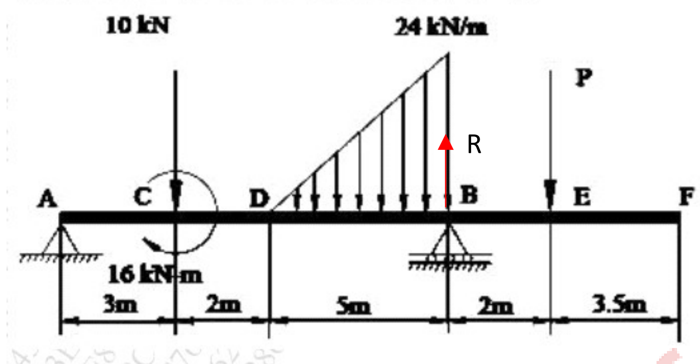
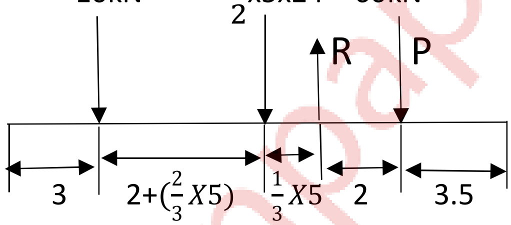


Figure 3

<u>Solution</u> :



$\sum Fx = 0$

$\sum Fy = 10 - 60 + R - P = 0$

$R - P = 50$ ……….(1)

$\sum M_B^F = (Px2) - (60x1.667) - (10x7) = 0$

$P = 85.01$ kN ……..(2)

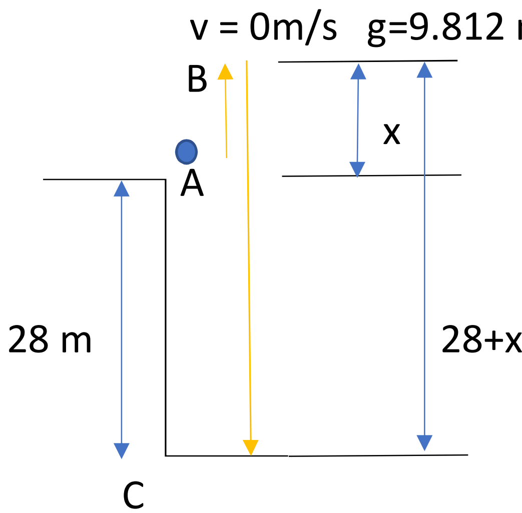
From (1) and (2)

$R = 135.01$ kN

The magnitudes of force P and reaction R are 85.01kN and 135.01kN respectively.

d) From the top of a tower, 28 m high, a stone is thrown vertically up with a velocity of 9m/s. After how much time will the stone reach the ground? With what velocity does it strike the ground? (5 marks)

<u>Solution</u> :



From A to B,

$u=9m/s; v=0m/s; a=-g=-9.812m/s^2$

$v = u + at_1$

$0 = 9 + (-9.812 \times t_1)$

$t_1 = 0.917$ sec

Now, $v^2 = u^2 + 2(-g)x$

$x = 4.128$ m

From B to C,

$u=0m/s$ $g=9.912m/s^2$ $s=x+28=32.128m$

$v^2=u^2+2gs$

$v=\sqrt{2 X 9.812 X 32.128} = 25.11m/s$

$v=u+gt_2$

$t_2 = \frac{v}{g}$

$t_2 = 2.559$ sec

Total time = $t_1 + t_2$

Total time = 0.917 + 2.559 = 3.476 sec

The stone will strike the ground after 3.476 sec at a velocity of 25.11 m/s

e) For the truss shown in figure 4, find: (i) zero force members, if any (Justify your answer with FBD), (ii) support reactions at C and D. (5 marks)

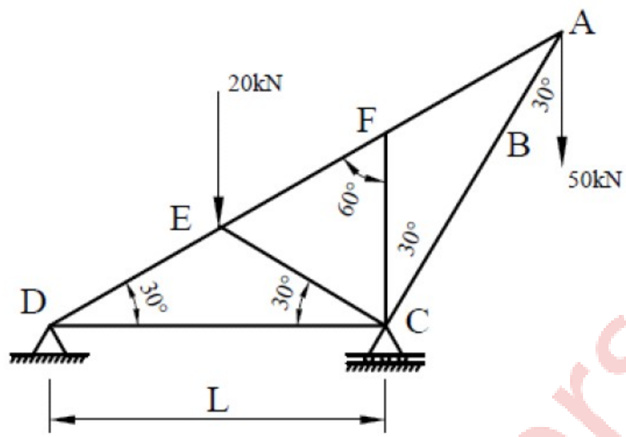
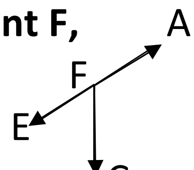


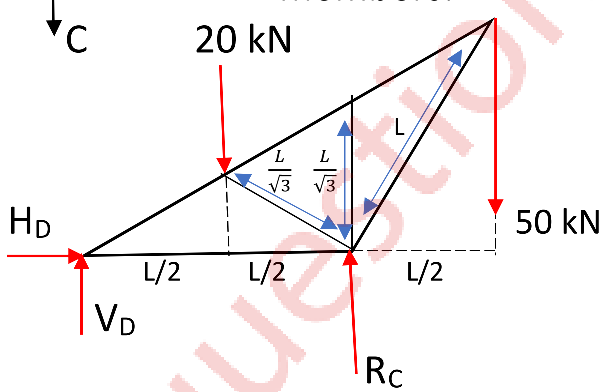
Figure 4

<u>**Solution**</u> **:**

At point F,



The member FC is a zero force member as there is no external load on F and there are two other collinear members.



$\sum F_x = 0$

$\boxed{H_D = 0}$

$\sum F_y = 0$

$-20 - 50 + R_C + V_D = 0$

$R_C + V_D = 70$ ....(1)

$\sum M_C^F = 0$

$+(V_D \times L) - (20 \times \frac{L}{2}) + (50 \times \frac{L}{2}) = 0$

$\boxed{V_D = -15 \text{ kN}}$

Substituting $V_D = -15$ kN in (1),

$\boxed{R_C = 85 \text{ kN}}$

Zero force members – FC

The magnitude of the support reactions at C and D are, $H_D = 0$ $V_D = -15$ kN and $R_C = 85$ kN respectively.

Q.2.

a) For the composite lamina shown in Figure 5, determine the coordinates of its centroid. (8 marks)

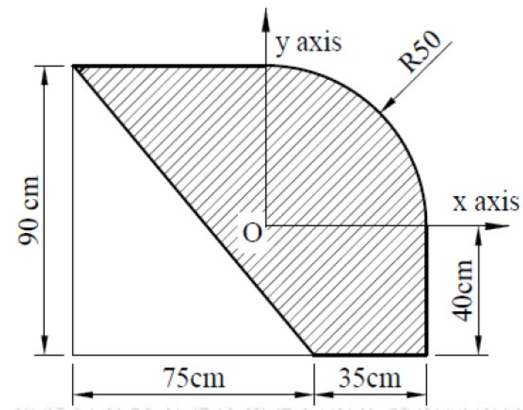


Figure 5

Solution :

Area of the shaded region = Rectangle ABFG + Rectangle OCDF + Quarter Circle OCB – Triangle AEF

Figure	Area	x coordinate	y coordinate	A_ix_i	A_iy_i
Rectangle ABFG	90 x 60 = 5400 mm^2	$\frac{-60}{2}$ = - 30	50 - $\frac{90}{2}$ = 5	-162000	27000
Rectangle OCDF	40 x 50 = 2000 mm^2	$\frac{50}{2}$ = 25	$-\frac{40}{2}$ = - 20	50000	-40000
Quarter Circle OCB	$\frac{1}{4}$ x π x 50^2 = 1963.495 mm^2	21.22	21.22	41665.3639	41665.3639
Triangle AEF	$-\frac{1}{2}$ x 75 x 90 = - 3375 mm^2	-35	-10	118125	33750

$\sum A_i$ = 5400 + 2000 + 1963.495 – 3375 = 5988.495

$\sum A_ix_i$ = -162000 + 50000 + 41665.3639 + 118125 = 47790.3639

$\sum A_iy_i$ = 27000 - 40000 + 41665.3639 + 33750 = 62415.3639

$\bar{x} = \frac{\sum Aixi}{\sum Ai} = \frac{3790.3639}{5988.495}$ = 7.98 m

$\bar{y} = \frac{\sum Aiyi}{\sum Ai}$ = 10.423 m

Coordinates of centroid are (7.98,10.423).

b) Replace the force system shown in Figure 6 with a single force and couple system acting at point B. (5 marks)

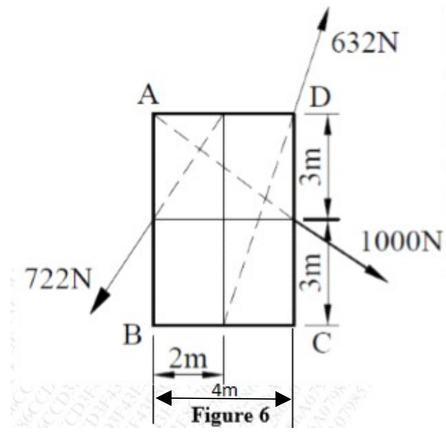
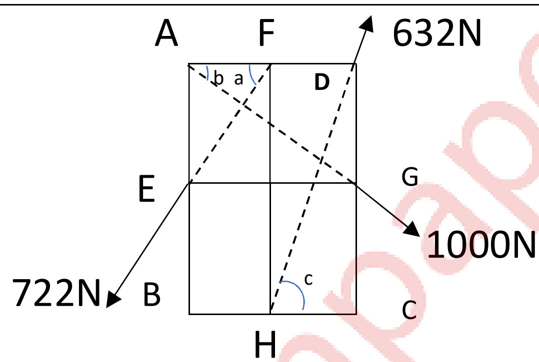


Figure 6

Solution :



In ΔAGF	In ΔAEF	In ΔFHD
$b=\tan^{-1}\frac{GD}{DA} = \tan^{-1}\frac{3}{4} = 36.87^0$	$a=\tan^{-1}\frac{AE}{AF} = \tan^{-1}\frac{3}{2} = 56.32^0$	$c=\tan^{-1}\frac{DC}{CH} = \tan^{-1}\frac{6}{2} = 71.57^0$

$\sum Fx = 1000\cos(36.87) + 632\cos(71.57) - 722\cos(56.32) = 599.42$

$\sum Fy = -1000\sin(36.87) + 632\sin(71.57) - 722\sin(56.32) = -601.23$

$R = \sqrt{Fx^2 + Fy^2}$

$= \sqrt{599.42^2 + (-601.23)^2}$

R = 848.99 N

$\Theta = \tan^{-1}(\frac{\sum Fy}{\sum Fx})$

$= \tan^{-1}(\frac{-601.23}{599.42})$

$= -45.086^0$

$\sum M_B^F = -[722\cos(56.32)x3] + [1000\cos(36.87)x6] - [632\sin(71.57)x2]$

$\sum M_B^F = 2399.66$ N-m

The magnitudes of the resultant force and couple at B are 848.99 N and 2399.66 N-m clockwise.

c) **The link CD of the mechanism shown in Figure 7 is rotating in counterclockwise direction at an angular velocity of 5 rad/s. For the given instance, determine the angular velocity of link AB.** (7 marks)

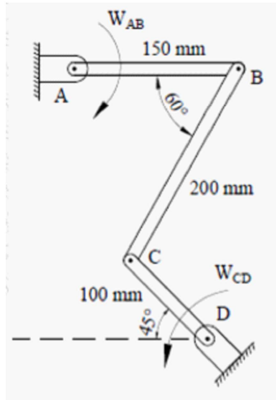


Figure 7

<u>Solution</u> :

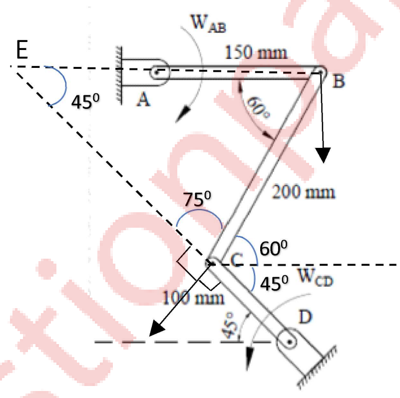


Figure 7

In ΔEBC,

Using Sine rule,

$$\frac{EB}{\sin(75)} = \frac{BC}{\sin(45)} = \frac{CE}{\sin(60)}$$

EB = $\frac{200\ X\ \sin(75)}{\sin(45)} = 0.27$ m CE = $\frac{200\ X\ \sin(60)}{\sin(45)} = 0.24$ m

$\omega_{CD} = V_C$ x CD

$V_C = \frac{5}{0.1}$ = 50 m/s

$\omega_{BC} = V_C$ x CE = 50 x 0.24 = 12 rad/s

$\omega_{BC} = V_B$ x EB

$V_B = \frac{12}{0.27}$ = 44.44 m/s

$\omega_{AB} = V_B$ x AB = 44.44 x 0.15 = 6.666 rad/s

The angular velocity of link AB is 6.666 rad/s.

Q.3.

a) **Cylinder A (diameter 1m, weight 20 kN) and cylinder B (diameter 1.5m, weight 40 kN) are arranged as shown in Figure 8. Find the reactions at all contact points. All contacts are smooth. (6 marks)**

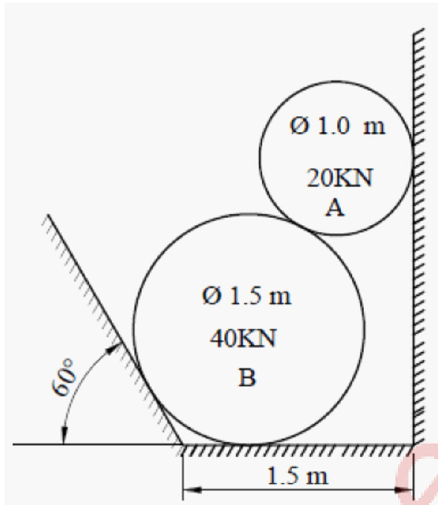


Figure 8

<u>Solution</u> :

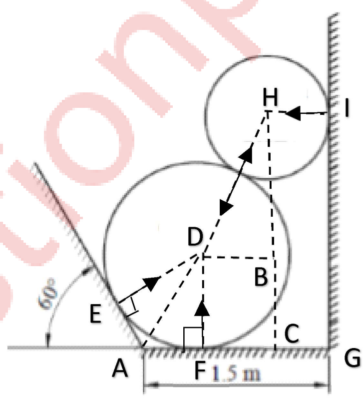


Figure 8

In ΔDAE and ΔDAF,

DA = DA;

∠DEA = ∠DFE;

By RHS rule , ΔDAE is congruent to ΔDAF

∠DAE = ∠DAF

∠DAE + ∠DAF = 2∠DAE = 180 – 60

∠DAE = ∠DAF = 60^0

In ΔDAF,

Tan(60) = $\frac{DF}{AF}$

AF = $\frac{0.75}{\tan(60)}$ = 0.433 m

AG = AF + FC + CG

1.5 = 0.433 + DB + HI

1.5 – 0.433 – HI = DB

1.5 – 0.433 – 0.5 = DB

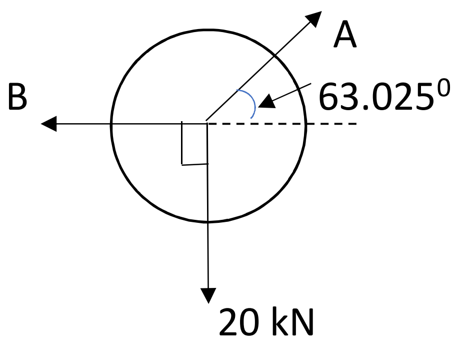
DB = 0.567 m

In ΔDBH,

Cos(D) = $\frac{DB}{DH}$ = $\frac{0.567}{0.75+\ .5}$ = 0.4536

D = $\cos^{-1} 0.4536$ = 63.025^0

Considering cylinder A,



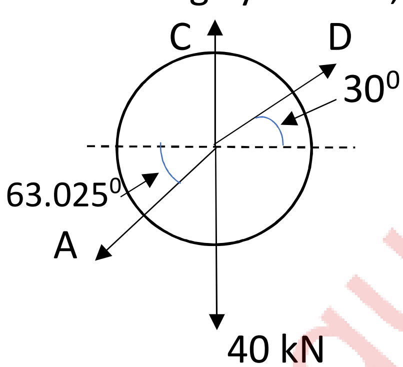
Using Lami's theorem,

$$\frac{20}{\sin(180-63.025)} = \frac{A}{\sin(90)} = \frac{B}{\sin(90+63.025)}$$

A = 22.44 kN

B = 10.1795 kN

Considering cylinder B,



∑Fx = 0

Dcos(30) – A cos(63.025) = 0 .....(1)

D = 11.75 kN

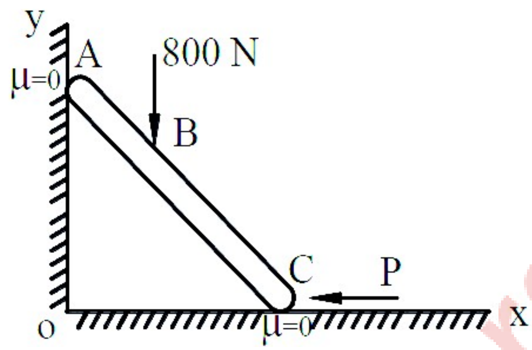
∑Fy = 0

C – 40 + Dsin(30) – Asin(63.025) = 0

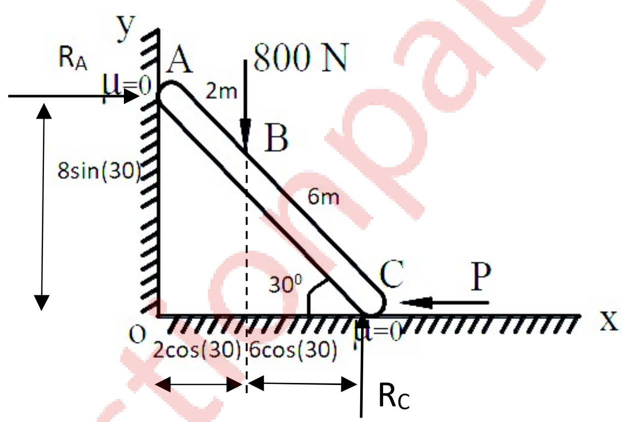
C =25.876 kN

The magnitudes of the reaction forces are
A = 22.44 kN
B = 10.1795 kN
C = 25.876 kN
D = 11.75 kN

b) Using Principle of Virtual Work, determine the force P which will keep the weightless bar AB in equilibrium. Take length AB as 2m and length AC as 8m. The bar makes an angle of 30^0 with horizontal. All the surfaces in contact are smooth. Refer Figure 9. **(6 marks)**



Solution :



Active force	Co-ordinate of point of application	Virtual displacement
-P	8cos(30)	-8sin(30)dθ
-800	6sin(30)	6cos(30)dθ

By principle of Virtual Work,

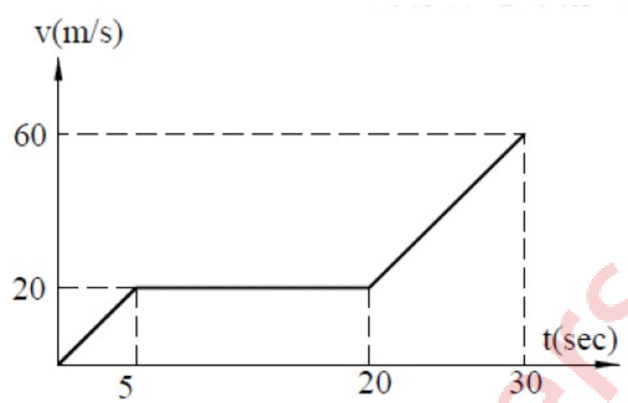
∑W = 0

-P(-8sin(30)dθ)-800(6cos(30)dθ)=0

P = 1039.23N

The magnitude of the force P is 1038.23N.

c) Velocity-time diagram for a particle travelling along a straight line is shown in Figure 10. Draw acceleration-time and displacement-time diagram for the particle. Also find important values of acceleration and displacement.
(8 marks)



<u>Solution</u> :

From 0 to 5 sec, u=0

v = at

a = $\frac{v}{t}$ = $\frac{20}{5}$ = 4 m/s^2

s = $\frac{1}{2}$at^2

s = $\frac{1}{2}$ x 4 x 5^2

s = 50m

From 5 to 20 sec,

u = v = 20 m/s

a = ut

s = 20 x 15

s = 300

Total displacement after 20 sec = $S_{0\text{ to }5}$ + $S_{5\text{ to }20}$ = 300 + 50 = 350m

From 20 to 30 sec, u=20m/s v=60m/s

v = u+at

a = $\frac{v-u}{t}$ = $\frac{60-20}{10}$ = 4 m/ s^2

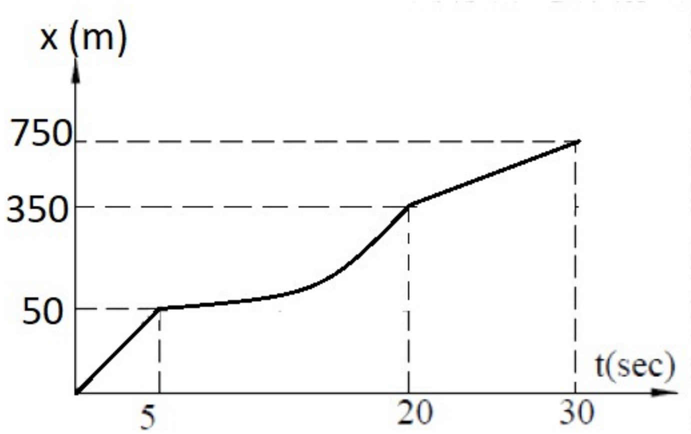
s = ut + $\frac{1}{2}$at^2

s = (20x10) + $\frac{1}{2}$ x 4 x 10^2

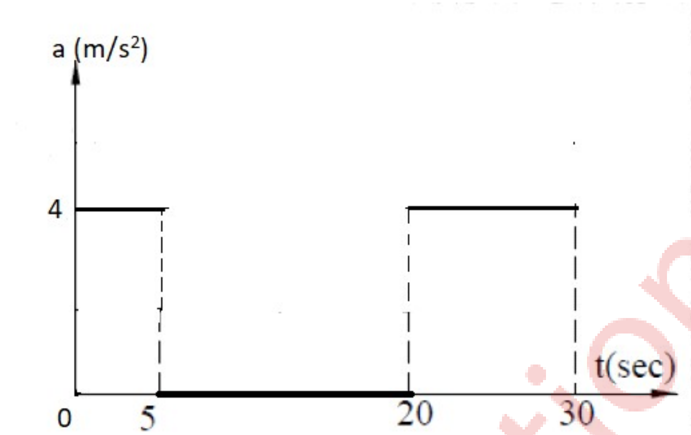
s = 400m

Total displacement after 30 sec = $S_{0 \text{ to } 20} + S_{20 \text{ to } 30}$ = 350 + 400 = 750m

x-t graph

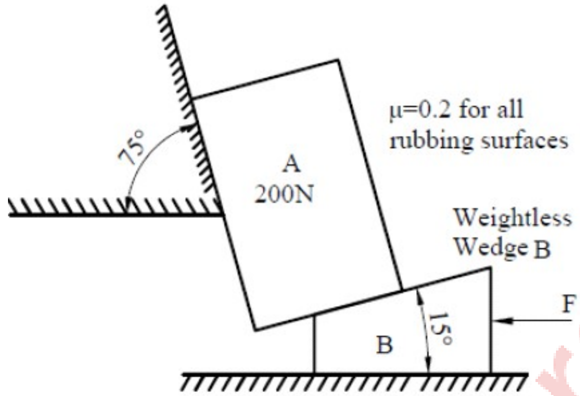


v-t graph

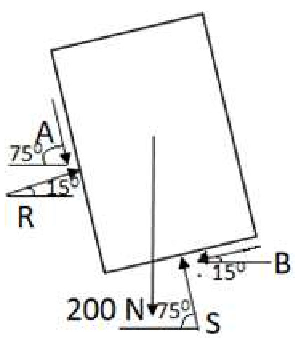


Q.4.

a) Find the force 'F' to have motion of block A impeding up the plane. Take coefficient of friction for all the surfaces in contact as 0.2. Consider the wedge B as weightless. Refer Figure 11. (7 marks)



<u>Solution</u> :



Consider block A,

$\sum Fx = 0$

$-B\cos(15) - S\cos(75) + A\cos(75) + R\cos(15) = 0$

$-0.2S\cos(15) - S\cos(75) + 0.2R\cos(75) + R\cos(15)$

$S(-0.2\cos(15)-\cos(75)) + R(0.2\cos(75)+\cos(15)) = 0$ ......(1)

$\sum Fy = 0$

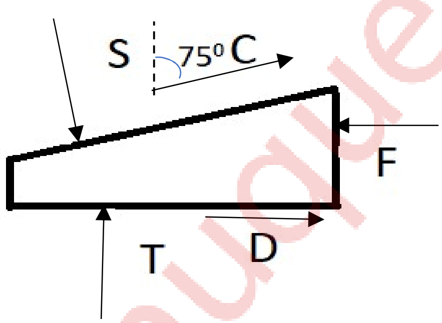
$-200 - B\sin(15) + S\sin(75) + R\sin(15) - A\sin(75) = 0$

$-0.2S\sin(15) + S\sin(75) - 0.2R\sin(75) + R\sin(15) = 200$

$S(-0.2\sin(15)+\sin(75)) + R(\sin(15)-0.2\sin(75) = 200$ ......(2)

Now, Solving (1) and (2),

S=212.019 N R=94.168 N



Consider block wedge B,

$\sum Fx = 0$

$-F + S\cos(75) + C\sin(75) + D = 0$

$F = S\cos(75) + 0.2S\sin(75) + 0.2T$

$\sum Fy = 0$

$T - S\sin(75) + C\cos(75) = 0$

$T = S\sin(75) - 0.2S\cos(75)$

T = 193.8197 N

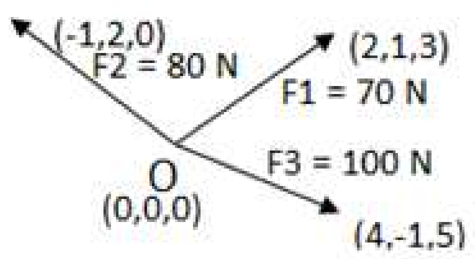
$F = S\cos(75) + 0.2S\sin(75) + 0.2TN$

F = 134.597 N

The magnitude of the force F is 134.597 N.

b) **Three forces F1, F2 and F3 act at the origin of Cartesian coordinate axes system. The force F1 (= 70N) acts along OA whereas F2 (= 80N) acts along OB and F3 (= 100N) acts along OC. The coordinates of the points A, B and C are (2,1,3), (-1,2,0) and (4,-1,5) respectively. Find the resultant of this force system.** **(5 marks)**

Solution :



$\overline{F1} = 70[\frac{2i+j+3k}{\sqrt{2^2+1^2+3^2}}]$ = 37.416 i + 18.708 j + 56.12 k

$\overline{F2} = 80[\frac{-i+2j}{\sqrt{-1^2+2^2}}]$ = -35.777 i + 71.554 j

$\overline{F3} = 100[\frac{4i-j+5k}{\sqrt{4^2+-1^2+5^2}}]$ = 61.721 i - 15.43 j + 77.152 k

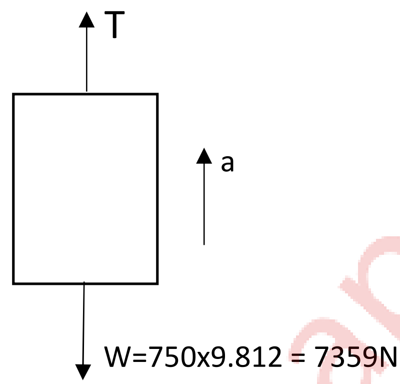
Resultant = $\overline{F} = \overline{F1} + \overline{F2} + \overline{F3}$ = 37.416 i + 18.708 j + 56.12 k - 35.777 i + 71.554 j + 61.721 i - 15.43 j + 77.152 k

Resultant = 63.36 i + 74.823 j + 133.272 k

The resultant of the force system = 63.36 i + 74.823 j + 133.272 k

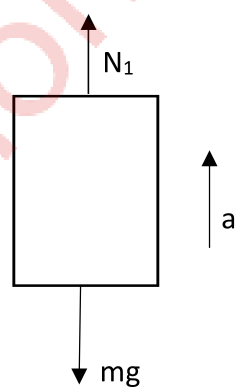
c) A 75kg person stands on a weighing scale in an elevator. 3 seconds after the motion starts from rest, the tension in the hoisting cable was found to be 8300N. Find the reading of the scale, in kg during this interval. Also find the velocity of the elevator at the end of this interval. The total mass of the elevator, including mass of the person and the weighing scale, is 750kg. If the elevator is now moving in the opposite direction, with same magnitude of acceleration, what will be the new reading of the scale? (8 marks)

Solution :

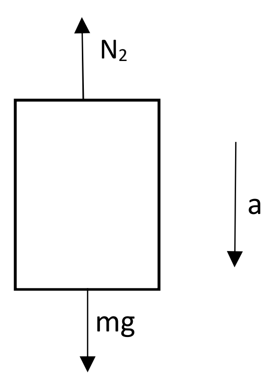


t = 3 sec
u = 0 m/s
T = 8300 N
$\sum F = ma$
T – W = 750 x a
8300 – 7359 = 750 x a
a = 1.255 m/s^2

v = u + at
v = 0 + (1.255 x 3)
v = 3.765 m/s



For upward motion,
$N_1 - mg = ma$
$N_1 = ma + mg$
$N_1 = m(a+g)$
$N_1 = 75(1.255+9.812)$
$N_1 = 830.025$ N
$N_1 = 84.59$ kg



For downward motion,
$N_2 - mg = - ma$
$N_2 = mg - ma$
$N_2 = m(g-a)$
$N_2 = 75(9.812 - 1.255)$
$N_2 = 641.775$ N
$N_2 = 65.407$ kg

In upward motion the reading on the weighing scale is 84.59 kg, final velocity at the end = 3.765 m/s and the reading on the weighing scale is 65.407 kg in the downward direction.

Q.5.

a) The cylinder B, diameter 400mm and weight 5kN, is held in position as shown in Figure 12 with the help of cable AB. Find the tension in the cable and the reaction developed at contact C. **(4 marks)**

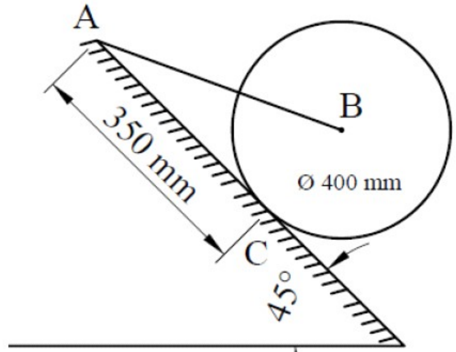


Figure 12

Solution :

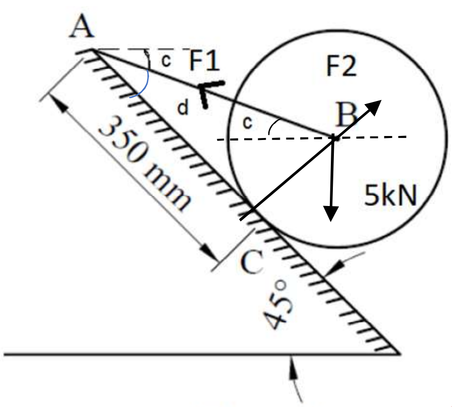


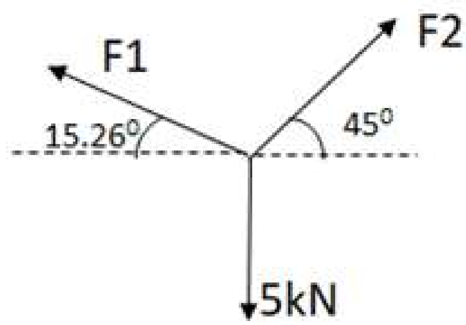
Figure 12

$\tan(d) = \frac{200}{350}$

$d = 29.74^0$

$c + d = 45^0$

$c = 15.26$



Using Lami's theorem,

$$\frac{5}{\sin(180-15.26-45)} = \frac{F1}{\sin(90+45)} = \frac{F2}{\sin(90+15.26)}$$

F1 = 4.072 kN

F2 = 5.555 kN

The magnitudes of the tension in the cable and the reaction developed at C are 4.072 kN and 5.555 kN.

b) Find the weight W_B so as to have its impending motion down the plane. Take weight of block A as 2kN. The pin connected rod AB is initially is in horizontal position. Refer Figure 13. Coefficient of friction = 0.25 for all surfaces. (5 marks)

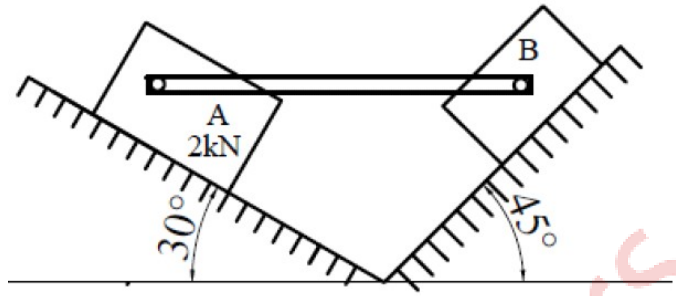
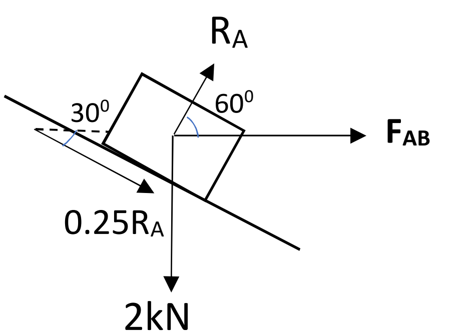
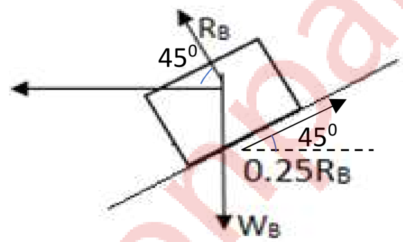


Figure 13

Solution :





A B

Consider block A

$\sum F_x = 0$

$F_{AB} + R_A\cos(60) + 0.25R_A\cos(30) = 0$

$F_{AB} + 0.7165R_A = 0$ ……(1)

$\sum F_y = 0$

$-2 + R_A\sin(60) - 0.25R_A\sin(30) = 0$

$0.741R_A = 2$

$R_A = 2.699$ kN …..(2)

$F_{AB} = -0.7165R_A$

$F_{AB} = -1.934$ kN ……(3)

Considering block B,

$\sum F_x = 0$

$-F_{AB} - R_B\cos(45) + 0.25R_B\cos(45) = 0$

$F_{AB} = -0.53R_B$

$R_B = 3.649$ kN

$\sum F_y = 0$

$-W_B + R_B\sin(45) + 0.25R_B\sin(45)$

$W_B = 3.225$ kN

The weight of the block B is 3.225 kN.

c) **Two springs, each having stiffness of 0.6N/cm and length 20 cm are connected to a ball B of weight 50N. The initial tension developed in each spring is 1.6N. The arrangement is initially horizontal, as shown in Figure 14. If the ball is allowed to fall from rest, what will be its velocity at D, after it has fallen through a height of 15 cm? (5 marks)**

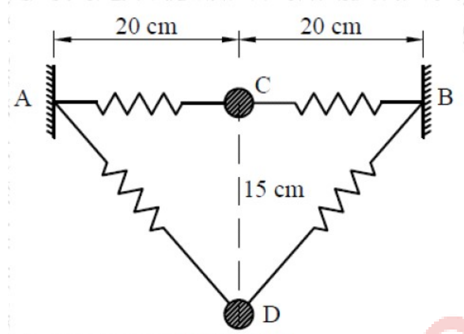


Figure 14

Solution :

Initial tension = 1.6 N

T = kx

1.6 = 0.6x

x_i = 2.667 cm ….(initial deformation)

Free length of the spring = l = 20 – x_i = 20 – 2.667 = 17.333 cm

The length of the spring at D = AD = $\sqrt{20^2 + 15^2}$ = 25 cm

Deformation at point D = x_f = 25 – 17.333 = 7.667 cm

Using work energy principle,

∑Work done = Change in K.E

Gravitational work + Spring work = $\frac{1}{2}m(V_D^2 - V_C^2)$

mgh +2 $[\frac{1}{2}k(x_i^2 - x_f^2)] = \frac{1}{2} \times 50 \times (V_D^2 - 0)$

$(50 \times 9.812 \times 15) + 0.6(2.667^2 - 7.667^2) = 25V_D^2$

$7359 - 31.002 = 25V_D^2$

$V_D^2 = 293.12$

$V_D = 17.12$ cm/s

The velocity of the ball at point D is 17.12 cm/s.

d) Two balls, A (mass 3kg) and B (mass 4kg), are moving with velocities 25 m/s and 40 m/s respectively (Refer Figure 15). Before impact, the direction of velocity of two balls are 300 and 500 with the line joining their centers as shown in Figure 15. If coefficient of restitution for the impact is 0.78, find the magnitude and the direction of velocities of the balls after the impact. (6 marks)

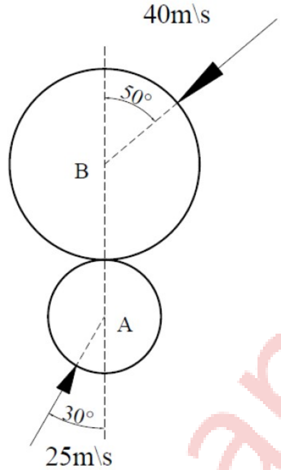
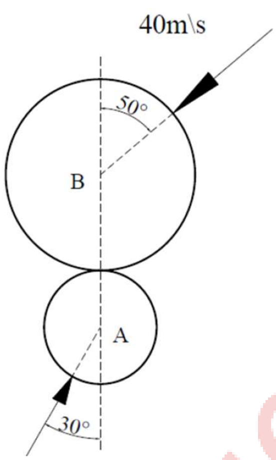


Figure 15

<u>Solution</u> :



$u_{Ax} = 25\sin(30) = 12.5$ m/s

$u_{Ay} = 25\cos(30) = 21.65$ m/s

$u_{Bx} = 40\sin(50) = 30.64$ m/s

$u_{By} = -40\cos(50) = -25.71$ m/s

Momentum is conserved only along the line of action.

$m_A u_{Ay} + m_B u_{By} = m_A v_{Ay} + m_B u_{By}$

$3(21.65) + 4(-25.71) = 3v_{Ay} + 4v_{By}$

$3v_{Ay} + 4v_{By} = -37.89$ .....(1)

$e = \frac{VBy - V}{UAy - UBy}$

$0.78 = \frac{VBy - VAy}{21.65 + 25.71}$

$v_{By} - v_{Ay} = 36.9408$ .....(2)

Solving (1) and (2)

$v_{Ay} = -26.52$ m/s $= 26.52$ m/s ↓

$v_{By} = 10.42$ m/s ↑

The magnitude of the velocities and direction of A and B are 26.52 m/s downwards and 10.42 m/s upwards respectively.

Q.6

a) For the truss shown in Figure 16, find the forces in members DE, BD and CB.
(5 marks)

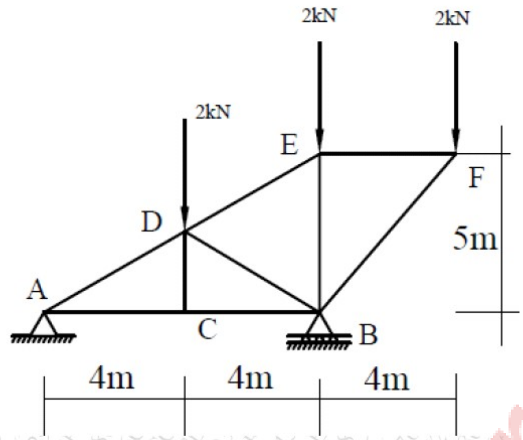
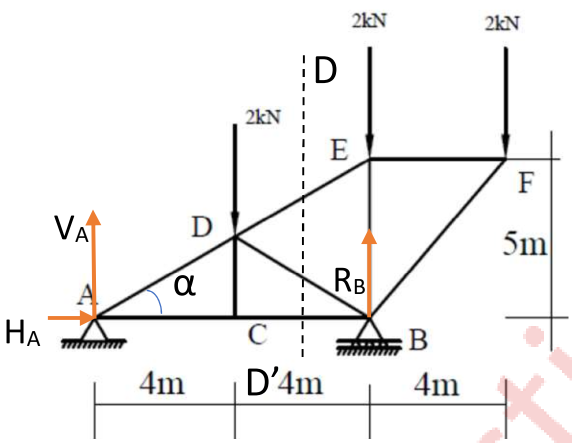


Figure 16

Solution :



$\sum Fx = 0$

$H_A = 0$

$\sum Fy = 0$

$V_A - 2 - 2 - 2 + R_B = 0$

$V_A + R_B = 6$ ....(1)

$\sum M_A{}^F = 0$

$(2x4) + (2x8) + (2x12) - (R_Bx8) = 0$

$R_B = 6$ kN

Now,

$V_A + R_B = 6$

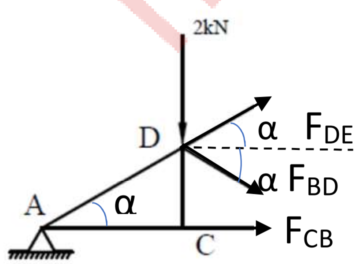
$V_A = 6 - 6 = 0$ kN

In ΔABE,

$\text{Tan}(\alpha) = \frac{EB}{AB} = \frac{5}{8}$

$\alpha = \tan^{-1} 0.625 = 32^0$

Taking section DD',



$\sum Fx = 0$

$F_{DE}\cos(32) + F_{BD}\cos(32) + F_{CB} = 0$ ......(1)

$\sum Fy = 0$

$-2 + F_{DE}\sin(32) - F_{BD}\sin(32) = 0$

$F_{DE}\sin(32) - F_{BD}\sin(32) = 2$ ....(2)

$\sum M_D^F = 0$

F_{CB} x perpendicular distance of F_{CB} from D = 0

F_{CB} = 0 kN…..(3)

Solving (1), (2) and (3),

F_{DE} = 1.887 kN

F_{BD} = - 1.887 kN

F_{CB} = 0 kN

The forces in the members DE, BD and CB are 1.887 kN (compression), 1.887 kN (tension) and 0 kN respectively.

b) A particle moves in x-y plane with acceleration components $a_x = -3 m/s^2$ and $a_y = -16t\ m/s^2$. If its initial velocity is V_0 = 50m/s directed at 35^0 to the x–axis, compute the radius of curvature of the path at t = 2 sec. (6 marks)

<u>Solution</u> :-

At t=0

V_0 = 50 m/s at 35^0 to the x-axis

V_x = 50cos(35) = 40.96 m/s

V_y = 50sin(35) = 28.68 m/s

Given, $a_x = -3\ m/s^2$ and $a_y = -16t\ m/s^2$

Integrating, $V_x = -3t + c_1$ and $V_y = -8t^2 + c_2$

At t=0

c_1 = 40.96 and c_2 = 28.68

Now,

$V_x = -3t + 40.96$ and $V_y = -8t^2 + 28.68$

At t=2sec

$V_x = -3(2) + 40.96$ and $V_y = -8(2^2) + 28.68$

V_x = 34.96 m/s and V_y = -3.32 m/s

$a_x = -3\ m/s^2$ and $a_y = -32\ m/s^2$

$V = \sqrt{Vx^2 + Vy^2} = \sqrt{34.96^2 + (-3.32)^2)} = 35.12$ m/s

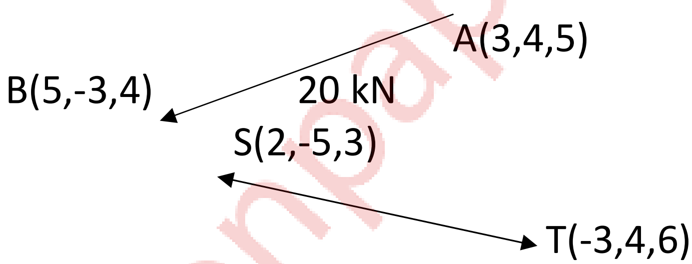
Radius of curvature at t = 2sec,

$$R = \frac{V^3}{|Vxay - Vyax|} = \frac{35.12^3}{|(34.96\ X - 32) - (-3.32\ X - 3)|} = 38.38 \text{ m}$$

The radius of curvature of the path at t = 2 sec is 38.38 m

c) A force of magnitude of 20kN, acts at point A(3,4,5)m and has its line of action passing through B(5,-3,4)m. Calculate the moment of this force about a line passing through points S(2,-5,3) m and T(-3,4,6)m. (5 marks)

Solution :



$$\overline{F1} = 20\left[\frac{(5-3)i + (-3-4)j + (4-5)k}{\sqrt{(5-3)^2 + (-3-4)^2 + (4-5)^2}}\right] = 5.44\ i - 19.05\ j - 2.72\ k \text{ kN}$$

$$\overline{M_S}^{F1} = \overline{SA} \times \overline{F1} = \begin{bmatrix} i & j & k \\ 3-2 & 4-(-5) & 5-3 \\ 5.44 & -19.05 & -2.72 \end{bmatrix} = 13.62\ i + 13.6\ j - 68.01\ k \text{ kN-m}$$

$$|M_S^{F1}| = \sqrt{(13.62)^2 + (-13.6)^2 + (-68.01)^2} = 70.68 \text{ kN-m}$$

$$\widehat{ST} = \frac{\overline{ST}}{|\overline{ST}|} \frac{(-3-2)i + (4+5)j + (6-3)k}{\sqrt{(-3-2)^2 + (4+5)^2 + (6-3)^2}} = -0.466\ i + 0.839\ j + 0.28\ k$$

Moment about the line,

$M_{ST}^{F1} = M_S^{F1} . \widehat{ST} = (13.62\ i + 13.6\ j - 68.01\ k).(-0.466\ i + 0.839\ j + 0.28\ k)$

$= -6.35 + 11.41 - 19.04$

$M_{ST}^{F1} = -13.98$ kN-m

Vector form,

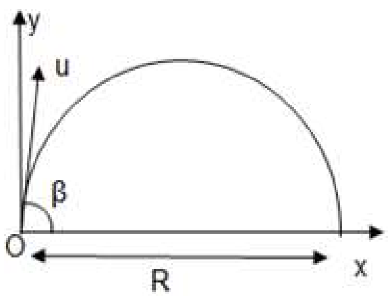
$\overline{M_{ST}}^{F1} = M_{ST}^{F1} . \widehat{ST} = -13.98(-0.466\ i + 0.839\ j + 0.28\ k)$

M_{ST}^{F1} = 6.51 i – 11.73 j – 3.91 k

The moment of the force about a line passing through points S(2,-5,3) m and T(-3,4,6)m is -13.98 kN-m (magnitude) and 6.51 i – 11.73 j – 3.91 k (vector form).

d) Find an expression for maximum range of a particle which is projected with an initial velocity of '*u*' inclined at an angle of 'β' with the horizontal. (4 marks)

Solution :



Consider a particle performing projectile motion.

R – Horizontal Range

T – Total flight time

Considering vertical components of motion,

$s = ut + \frac{1}{2}at^2$

$0 = u\sin(\beta) - \frac{1}{2}gT^2$

$$T = \frac{2u\sin(\beta)}{g}$$

Considering horizontal components of motion,

$s = ut + \frac{1}{2}at^2$

R = ucos(β)T + 0 …….(as acceleration in x direction is zero)

$R = u\cos(\beta) \times \frac{2u\sin(\beta)}{g}$

$R = \frac{u^2 \sin(2\beta)}{g}$

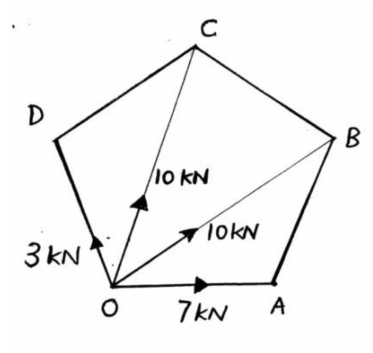
For maximum Range, sin(2β) should be maximum, i.e. sin(2β) = 1, i.e. 2β = 90, i.e. $\beta = 45^0$

$R_{max} = \frac{u^2}{g}$

Mumbai University Engineering Mechanics May 2019 Question Paper Solution

Q1. Attempt Any Four:

(a) Find the resultant of forces as shown in fig. **(05 marks)**



Solution:

$$\sum F_x = \left[(7)+(10\cos(36°))+(10\cos(72°))+(-3\cos(72°))\right]\ \text{kN}$$

$$\therefore \sum F_x = 17.25\ \text{kN}(\rightarrow)$$

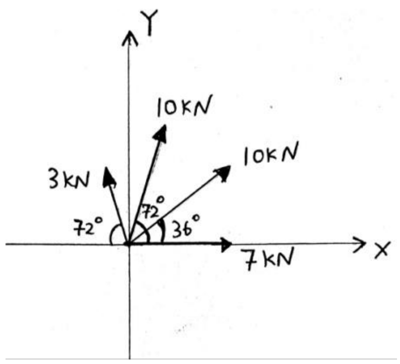
$$\sum F_y = \left[(10\sin(36°))+(10\sin(72°))+(3\sin(72°))\right]\ \text{kN}$$

$$\therefore \sum F_y = 18.24\ \text{kN}(\uparrow)$$

$$\text{Resultant}=\sqrt{\left(\sum F_x\right)^2+\left(\sum F_y\right)^2}\ \text{kN}$$

$$=\sqrt{(17.25)^2+(18.24)^2}\ \text{kN}$$

$$=25.10\ \text{kN}\ (□)$$

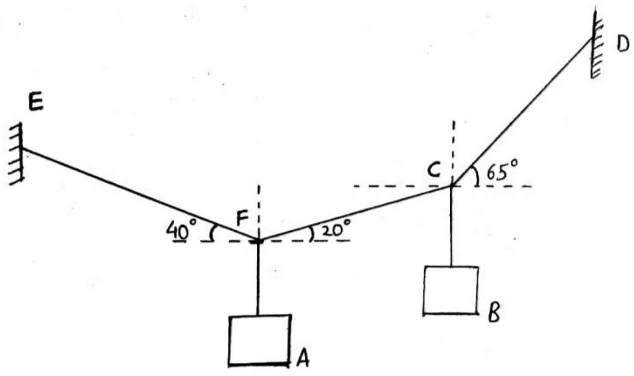


$$\therefore \boxed{\text{Resultant}=25.10\ \text{kN}\ (□)}$$

$$\theta=\tan^{-1}\left[\frac{\sum F_y}{\sum F_x}\right]=\tan^{-1}\left[\frac{18.24}{17.25}\right]=46.6°$$

$$\therefore \boxed{\theta = 46.6°}$$

(b) If the cords suspended the two buckets in equilibrium position shown in Fig. Determine weight of bucket B if bucket A has a weight of 60 N. (05 marks)



Solution:

$W_A = 60 \text{ N} \quad ...\{\text{Given}\}$

Applying Lami's Theorem at point F,

$$\therefore \frac{W_A}{\sin(120°)} = \frac{F_{FC}}{\sin(130°)} = \frac{F_{FE}}{\sin(110°)}$$

$$\therefore F_{FC} = \frac{W_A}{\sin(120°)}(\sin(130°)) \text{ N} = \frac{60}{\sin(120°)}(\sin(130°)) \text{ N}$$

$$\therefore F_{FC} = 53.07 \text{ N}$$

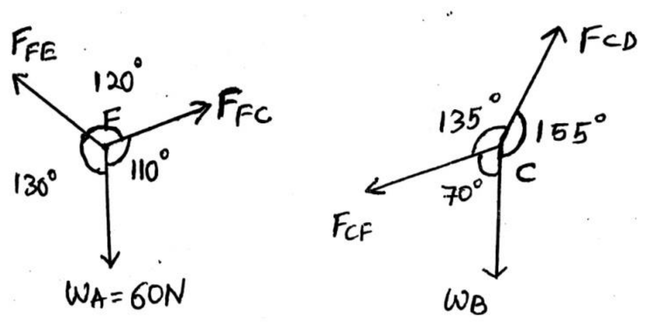
Applying Lami's Theorem at point C,

$$\therefore \frac{W_B}{\sin(135°)} = \frac{F_{CD}}{\sin(70°)} = \frac{F_{CF}}{\sin(155°)}$$

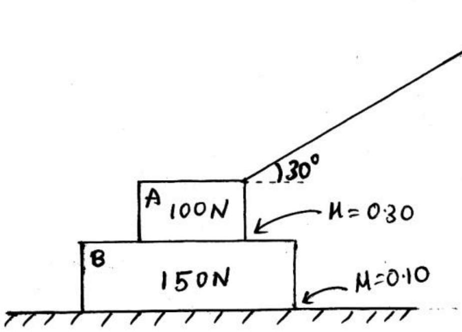
$$\therefore W_B = \frac{F_{CF}}{\sin(155°)}(\sin(135°)) \text{ N} = \frac{53.07}{\sin(155°)}(\sin(135°)) \text{ N}$$

$$\therefore W_B = 88.79 \text{ N}$$

$\therefore$ The weight of the bucket B = 88.79 N

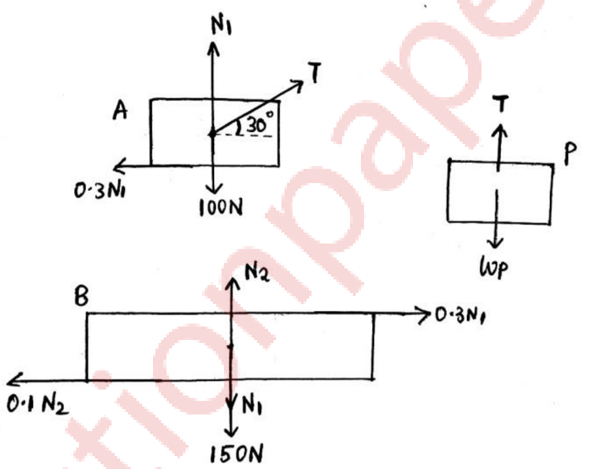


(c) Two blocks A=100 N and B=150 N are resting on the ground as shown in fig. Find the minimum weight P in the pan so that body A starts. Assume pulley to be mass less and frictionless. (05 marks)



Solution:

The FBD is shown below,



Let the tension in the string be 'T' N

Let the normal force between the two blocks A and B be N_1 N

Let the normal force between block B and ground be N_2 N

For block to just start to move, the friction force acting on block A will be backwards

And on block B the same force will be forwards.

The friction force between block B and ground will be backwards on block B.

$\therefore$ Applying equilibrium conditions on Block B,

$$\sum F_x = 0$$

$$\therefore (0.3N_1) - (0.1N_2) = 0 \qquad ...(1)$$

$\sum F_y = 0$

$\therefore N_2 - 150 - N_1 = 0$

$\therefore -N_1 + N_2 = 150 \qquad ...(2)$

From (1) and (2)

$N_1 = 75$ N and $N_2 = 225$ N

$\therefore$ Applying equilibrium conditions on Block A,

$\sum F_x = 0$

$\therefore (T\cos(30°))\text{-}(0.3N_1) = 0$

$\therefore (T\cos(30°))\text{-}(0.3(75)) = 0$

$\therefore$ T= 25.98 N

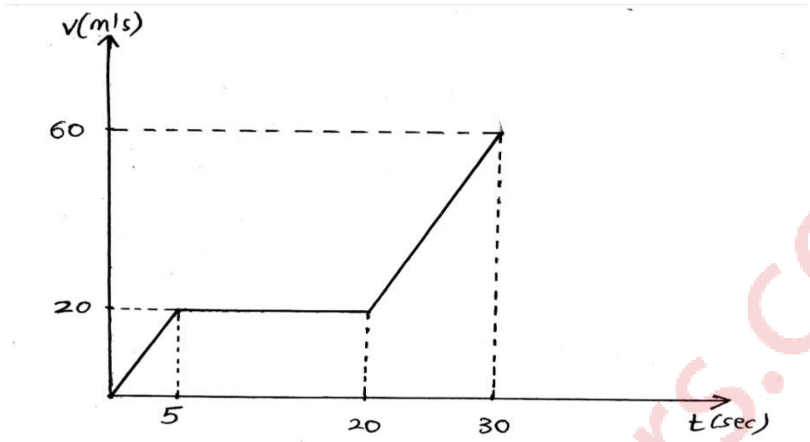
On block P,

$W_P = T$

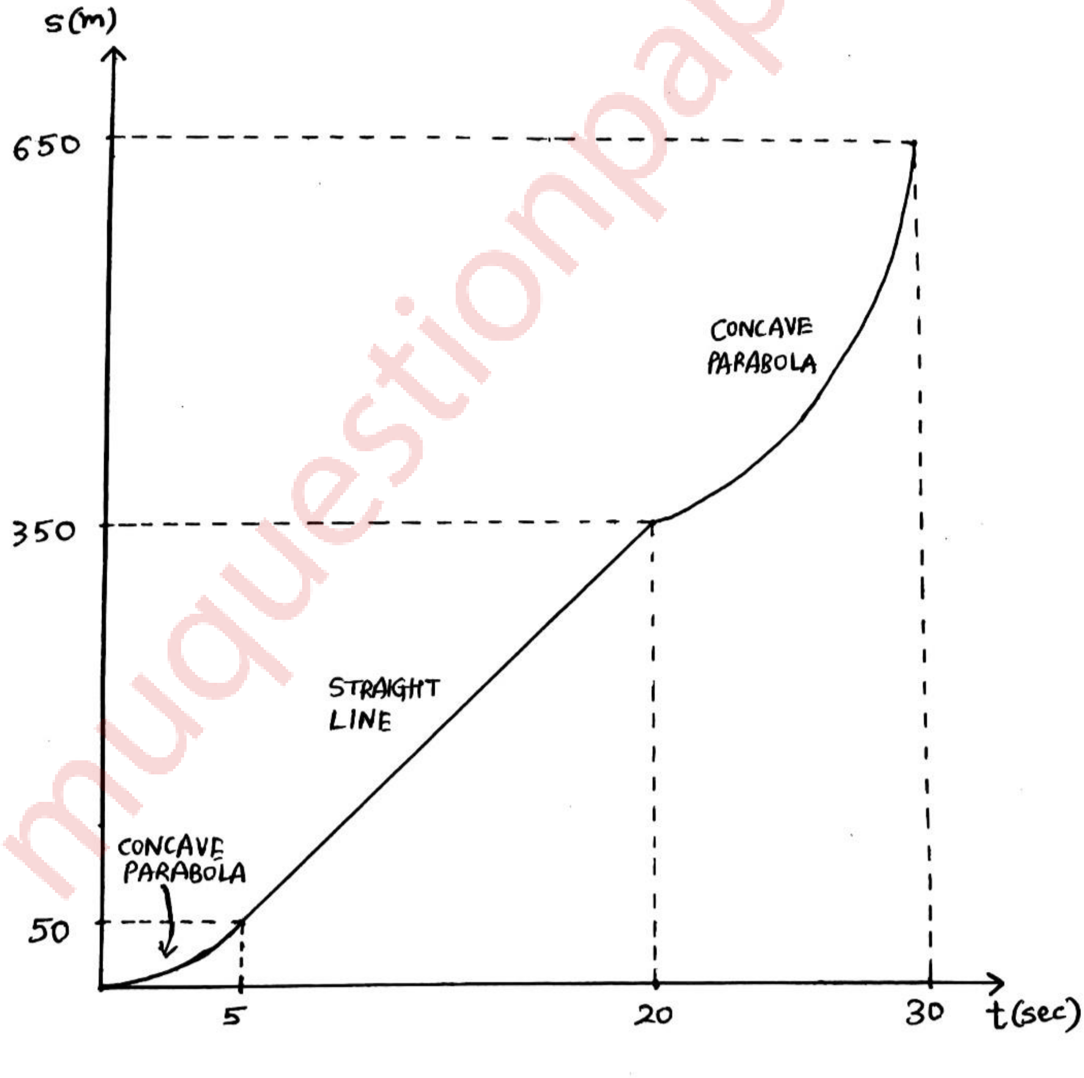
$\therefore$ The minimum weight P in the pan so that block A just starts = 25.98 N

(d) The motion of jet plane while travelling along a runway is defined by the v-t graph as shown in Fig. Construct the s-t graph for the motion. The plane starts from rest.

(05 marks)



Solution: The required graph is:



Explanation :

For S-T graph

$\int ds = \int v.dt$ =Area under the graph under time interval

Since the object is at rest initially, $S_0 = 0$ m

For time 0 to 5 seconds

$\int_0^{S_5} ds = \int_0^5 v.dt$ = Area under graph from 0 to 5 seconds= $\frac{1}{2}(5)(20)$ m

$\therefore S_5 - 0 = 50$ m $\quad \therefore \boxed{S_5 = 50 \text{ m}}$

$\int_{S_5}^{S_{20}} ds = \int_5^{20} v.dt$ = Area under graph from 5 to 20 seconds= $(20-5)(20)$ m

$\therefore S_{20} - S_5 = 300$ m $\quad \therefore \boxed{S_{20} = 350 \text{ m}}$

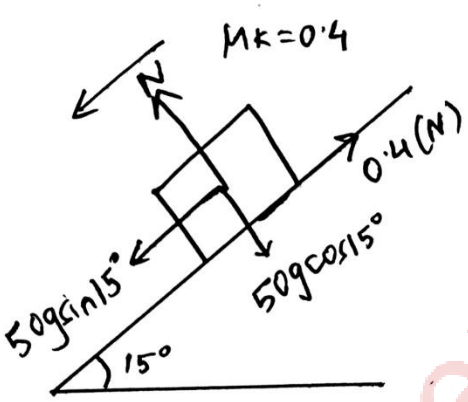
$\int_{S_{20}}^{S_{30}} ds = \int_{20}^{30} v.dt$ = Area under graph from 20 to 30 seconds= $\left[(30-20)(20)+\frac{1}{2}(30-20)(60-20)\right]$ m

$\therefore S_{30} - S_{20} = 400$ m $\quad \therefore \boxed{S_{30} = 750 \text{ m}}$

(e) A 50 kg block is kept on the top of a $15°$ slopping surface is pushed down the plane with an initial velocity of 20 m/s. If $\mu_k = 0.4$, determine the acceleration of the block.

(05 marks)

Solution: The FBD is,



On the block,

N=50gsin(15°)=50(9.8)sin(15°)

$\therefore N$=126.82 N

$50\text{gsin}(15°) - 0.4(N) = 50(a)$ where a is acceleration of the block

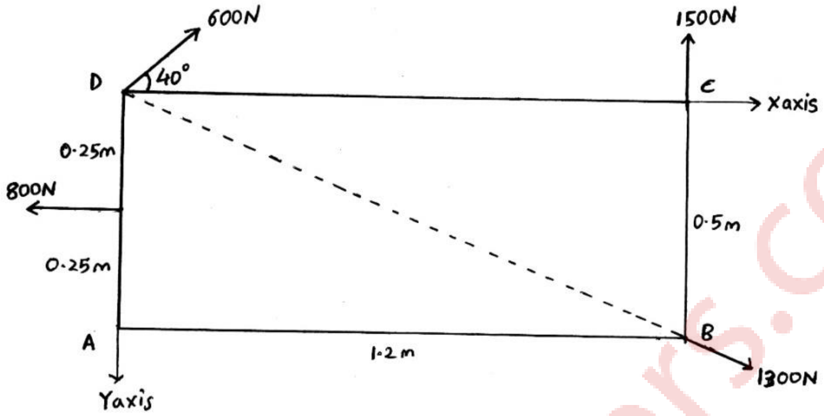
$50(9.8)\sin(15°) - 0.4(126.82) = 50(a)$

$\therefore a = 1.52 \text{ m/s}^2$

$\therefore$ **The acceleration of the block =** 1.52 m/s^2

Q2. Attempt:

(a) Four forces acting on a rectangle in the same plane as shown in fig. below. Find magnitude and direction of resultant force. Also find intersection of line of action of resultant with X and Y axes, assuming D as origin. (06 marks)



Solution:

Finding $\angle ABD$

$$\angle ABD = \tan^{-1}\left[\frac{AD}{AB}\right] = \tan^{-1}\left(\frac{0.5}{1.2}\right)$$

$\therefore \angle ABD = 22.62°$

$\therefore$ The angle made by 1300 N force with horizontal is 22.62°

$$\sum F_x = [1300\cos(22.62°) + 600\cos(40°) - 800] \text{ N}$$
$$\sum F_x = 859.63 \text{ N } (\rightarrow)$$

$$\sum F_y = [600\sin(40°) + 1500 - 1300\sin(22.62°)] \text{ N}$$
$$\sum F_y = 1385.67 \text{ N } (\uparrow)$$

$$\text{Resultant} = \sqrt{\left(\sum F_x\right)^2 + \left(\sum F_y\right)^2} \text{ N}$$
$$= \sqrt{859.63^2 + 1385.67^2} \text{ N}$$
$$= 1630.66 \text{ N } (\ )$$

$\therefore$ $\boxed{\text{Resultant} = 1630.66 \text{ N } (\ )}$

$$\theta = \tan^{-1}\left[\frac{\sum F_y}{\sum F_x}\right] = \tan^{-1}\left(\frac{1385.67}{859.63}\right) = 58.19°$$

$\therefore$ $\boxed{\theta = 58.19°}$

$$\sum M_0 = (1500)(1.2) + (1300\cos(22.62°)(0.5) - \left(1300\sin(22.62°)(1.2)\right) - (800)(0.25)$$

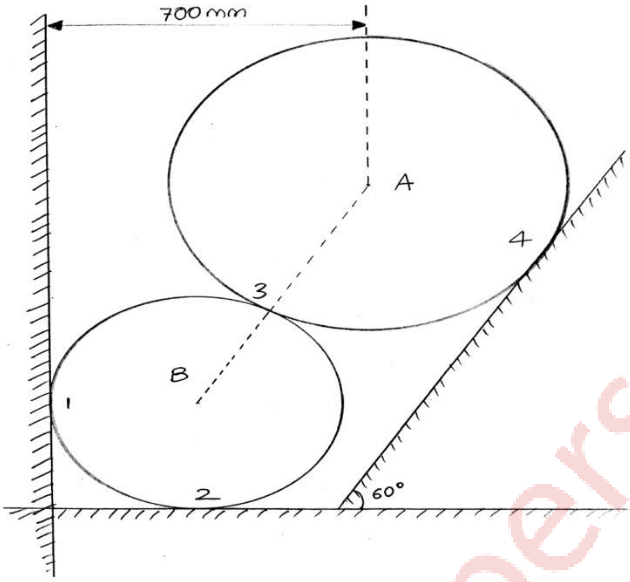
$$\therefore \sum M_0 = 1600 \text{ N-m}$$

$$X = \frac{\sum M_0}{\sum F_y} = \frac{1600}{1385.67} = 1.15 \text{ m}$$

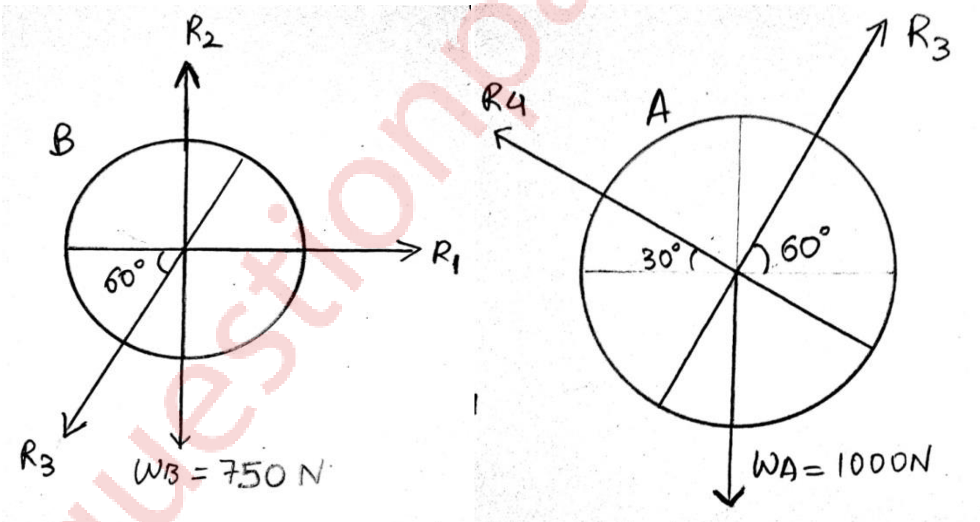
$$Y = \frac{\sum M_0}{\sum F_x} = \frac{1600}{859.63} = 1.86 \text{ m}$$

$$\therefore \boxed{X = 1.15 \text{ m and } Y = 1.86 \text{ m}}$$

(b) Two spheres A and B of weight 1000 N and 750 N respectively are kept as shown in fig. Determine the reactions at all contact points 1, 2, 3 and 4. Radius of A=400 mm and B=300 mm. **(08 marks)**



Solution:



Applying equilibrium conditions on ball A

$$\sum F_x = 0$$

$$\therefore R_3 \cos(60°) - R_4 \cos(30°) = 0 \quad ...(1)$$

$$\sum F_y = 0$$

$$\therefore R_3 \sin(60°) + R_4 \sin(30°) - 1000 = 0$$

$$\therefore R_3 \sin(60°) + R_4 \sin(30°) = 1000 \quad ...(2)$$

From (1) and (2)

$$\boxed{R_3 = 866.03 \text{ N and } R_4 = 500 \text{ N}}$$

Applying equilibrium conditions on ball B

$$\sum F_x = 0$$

$$\therefore R_1 - R_3\cos(60°)=0$$

$$\therefore R_1 = R_3\cos(60°) = (866.03)\cos(60°)$$

$$\therefore \boxed{R_1 = 433.02 \text{ N}}$$

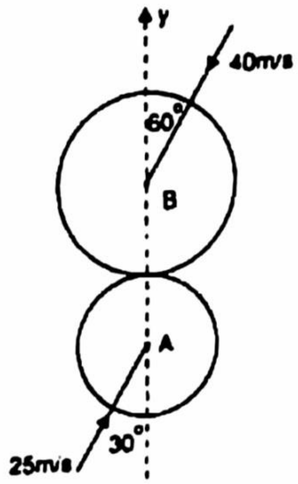
$$\sum F_y = 0$$

$$\therefore R_2 - R_3\sin(60°) - 750 = 0$$

$$R_2 - (866.03)\sin(60°) - 750 = 0$$

$$\therefore \boxed{R_2 = 1500 \text{ N}}$$

(c) Two smooth balls A (mass 3 kg) and B (mass 4 kg) are moving with velocities 25 m/s and 40 m/s respectively. Before impact, the directions of velocity of two balls are 30° and 60° with the line joining the centres as shown in Fig. If e=0.8, find magnitude and direction of velocities of the balls after impact. (06 marks)



Solution:

Let u_A and u_B be the initial velocities of balls A and B respectively,

Let v_A and v_B be the final velocities of balls A and B respectively,

$\therefore u_A = 25\sin(30°)i + 25\cos(30°)j$

$\therefore u_B = -40\sin(60°)i - 40\cos(30°)j$

Let v_{Ax} and v_{Ay} be the x and y components of velocity of ball A resp,

Let v_{Bx} and v_{By} be the x and y components of velocity of ball B resp,

Applying Law of conservation of linear momentum along y direction,

$m_A u_{Ay} + m_B u_{By} = m_A v_{Ay} + m_B v_{By}$

$\therefore 3(25\cos(30°)) + 4(-40\cos(60°)) = 3v_{Ay} + 4v_{By} \quad ...(1)$

$$e = \frac{v_{By} - v_{Ay}}{u_{Ay} - u_{By}} = 0.8 = \frac{v_{By} - v_{Ay}}{25\cos(30°) - (-40\cos(60°))}$$

$0.8\left[25\cos(30°) - (-40\cos(60°))\right] = -v_{Ay} + v_{By} \quad ...(2)$

From (1) and (2)

$v_{Ay} = -21.19$ m/s $v_{By} = 12.13$ m/s

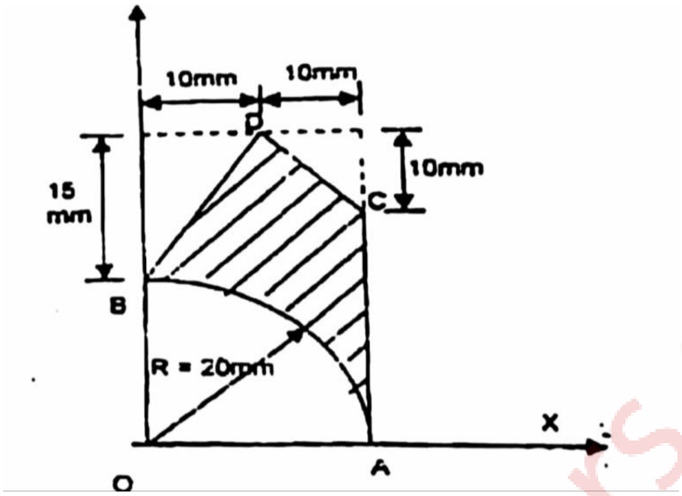
The velocities along the perpendicular to line of action remain unchanged,

$\therefore v_{Ax} = 25\sin(30°) = 12.5$ m/s and $v_{Bx} = -40\sin(60°) = -34.64$ m/s

$$\therefore \boxed{\begin{array}{l} v_A = [12.5i - 21.19j] \text{ m/s} \\ v_B = [-34.64i + 12.13j] \text{ m/s} \end{array}}$$

Q3. Attempt:

(a) Find the centroid of shaded area as shown in fig. (08 marks)



Solution:

	Shape	Area(in mm^2)	X-Coordinate	Y-Coordinate	AX	AY
1.	Rectangle	$=(35)(20)$ $=700$	$=10$	$=17.5$	$=7000$	$=12250$
2.	Quarter Circle	$=-\frac{\pi(20)^2}{4}$ $=-314.16$	$=\frac{4(20)}{3\pi}$ $=8.49$	$=\frac{4(20)}{3\pi}$ $=8.49$	$=-2667.22$	$=-2667.22$
3.	Triangle (ht=15 mm, bs=10 mm)	$=-\frac{1}{2}(10)(15)$ $=-75$	$=\frac{10}{3}$ $=3.33$	$=35-\frac{15}{3}$ $=30$	$=-250$	$=-2250$
4.	Triangle (ht=10 mm, bs=10 mm)	$=-\frac{1}{2}(10)(10)$ $=-50$	$=20-\frac{10}{3}$ $=16.67$	$=35-\frac{10}{3}$ $=31.67$	$=-833.5$	$=-1583.5$

$$\sum A = 700-314.16-75-50 \text{ mm}^2$$
$$\therefore \sum A = 260.84 \text{ mm}^2$$

$$\sum AX = 7000-2667.22-250-833.5 \text{ mm}^2$$
$$\therefore \sum AX = 3249.28 \text{ mm}^2$$

$$\sum AY = 12250-2667.22-2250-1583.5 \text{ mm}^2$$
$$\therefore \sum AY = 5749.28 \text{ mm}^2$$

$$X = \frac{\sum AX}{\sum A} = \frac{3249.28}{260.84} = 12.46 \text{ mm}$$

$$Y = \frac{\sum AY}{\sum A} = \frac{5749.28}{260.84} = 22.04 \text{ mm}$$

Centroid is,

$\therefore$ $\boxed{X=12.46 \text{ mm and } Y=22.04 \text{ mm}}$

(b) Three forces F_1, F_2 and F_3 act at origin O. $F_1 = 70$ N acting along OA, where A (2, 1, 3). $F_2 = 80$ N acting along OB, where B(-1, 2, 0). $F_3 = 100$ N acting along OC, where C(4, -1, 5). Find the resultant of these concurrent forces. **(06 marks)**

Solution:

$\overline{F_1} = 70[2i + j + 3k]$ N

$\overline{F_2} = 80[-i + 2j]$ N

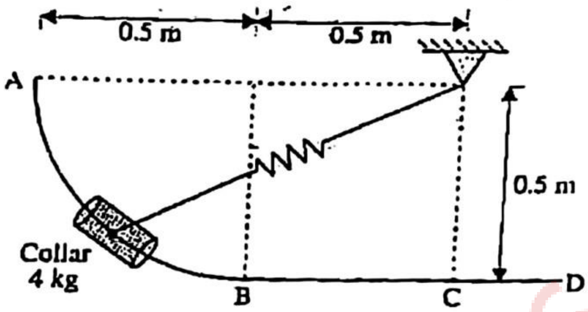
$\overline{F_3} = 100[4i - j + 5k]$ N

$\overline{F_{net}} = \overline{F_1} + \overline{F_2} + \overline{F_3}$ N

$\therefore \overline{F_{net}} = [70[2i + j + 3k] + 80[-i + 2j] + 100[4i - j + 5k]]$ N

$\therefore \overline{F_{net}} = [460i + 130j + 710k]$ N

(c) A 4 kg collar is attached to a spring, slides on a smooth bent rod ABCD. The spring has constant k=500 N/m and is undeformed when the collar is at 'C'. If the collar is released from rest at A. Determine the velocity of collar, when it passes through 'B' and 'C'. Also find the distance moved by collar beyond 'C' before it comes to rest again. Refer fig. (06 marks)



Solution:

$l(OB) = \sqrt{0.5^2 + 0.5^2} = 0.5\sqrt{2} = 0.707 \text{ m}$

Natural Length (l_0) =0.5 m

$x_A = OA - l_0 = 1 - 0.5 = 0.5 \text{ m}$
$x_B = OB - l_0 = 0.707 - 0.5 = 0.207 \text{ m}$

Applying work energy theorem from A to B

$W_g + W_{sp} = \Delta K$

$mgh + \frac{1}{2}k\left(x_A{}^2 - x_B{}^2\right) = \frac{1}{2}m\left(v_B{}^2 - v_A{}^2\right)$

$4(9.8)(0.5) + \frac{1}{2}(500)(0.5^2 - 0.207^2) = \frac{1}{2}(4)(v_B{}^2 - 0^2)$

$\therefore \boxed{v_B = 5.97 \text{ m/s}}$

Applying work energy theorem from B to C,

$W_g + W_{sp} = \Delta K$

$mgh + \frac{1}{2}k\left(x_B{}^2 - x_C{}^2\right) = \frac{1}{2}m\left(v_C{}^2 - v_B{}^2\right)$

$0 + \frac{1}{2}(500)(0.207^2 - 0^2) = \frac{1}{2}(4)(v_C{}^2 - 5.97^2)$

$\therefore \boxed{v_C = 6.40 \text{ m/s}}$

Applying work energy theorem from C to D,

$$W_g + W_{sp} = \Delta K$$

$$mgh + \frac{1}{2}k\left(x_C{}^2 - x_D{}^2\right) = \frac{1}{2}m\left(v_D{}^2 - v_C{}^2\right)$$

$$0 + \frac{1}{2}(500)(0^2 - x_D{}^2) = \frac{1}{2}(4)(0^2 - 6.4^2)$$

$$\therefore x_D = 0.572 \text{ m}$$

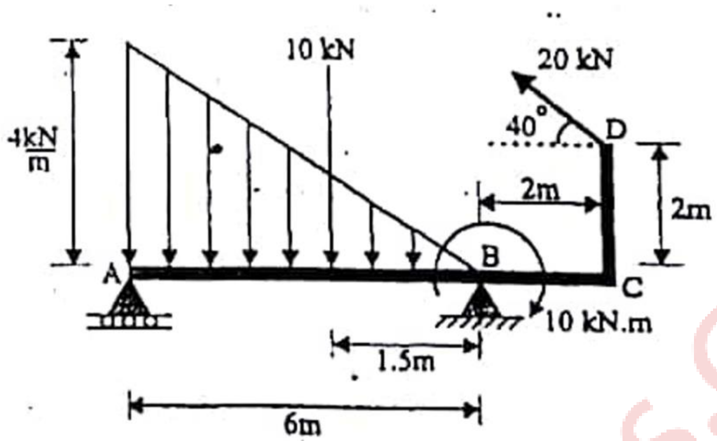
$$l(OD) = l_0 + x_D = 0.5 + 0.572 = 1.07 \text{ m}$$

$$\therefore \text{CD}=\sqrt{OD^2 - OC^2} = \sqrt{1.07^2 - 0.5^2} = 0.946 \text{ m}$$

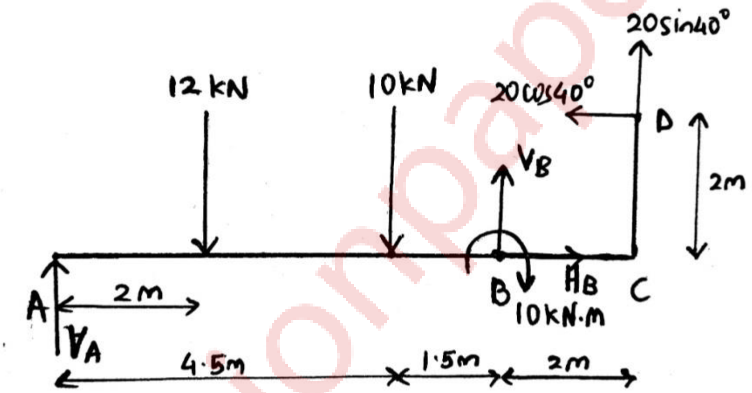
$$\therefore \boxed{\text{CD}=0.946 \text{ m}}$$

Q4. Attempt:

(a) Find the support reactions of beam loaded as shown in fig. (08 marks)



Solution: The FBD is,



$$\sum M_B = 0$$

$$\therefore -V_A(6) + 12(4) + 10(1.5) + 20\cos(40°)(2) + 20\sin(40°)(2) = 0$$

$$\therefore \boxed{V_A = 19.89 \text{ kN } (\uparrow)}$$

$$\sum F_X = 0$$

$$\therefore H_B - 20\cos(40°) = 0$$

$$\therefore \boxed{H_B = 15.32 \text{ kN } (\rightarrow)}$$

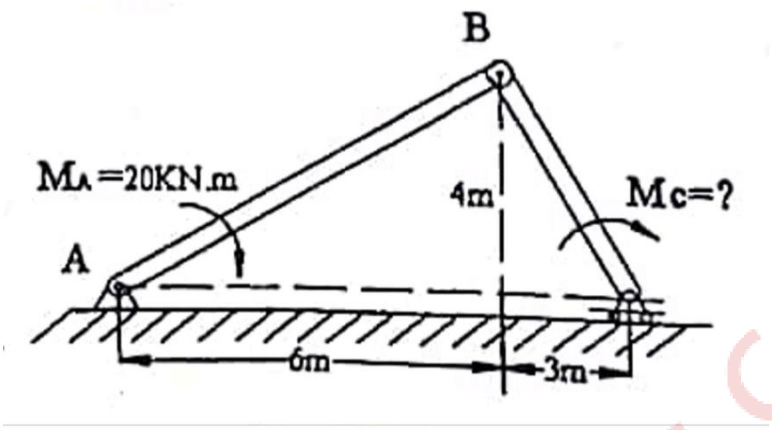
$$\sum F_Y = 0$$

$$\therefore V_A - 12 - 10 + V_B + 20\sin(40°) = 0$$

$$19.89 - 12 - 10 + V_B + 20\sin(40°) = 0$$

$$\therefore \boxed{V_B = -10.75 \text{ kN} = 10.75 \text{ kN} (\downarrow)}$$

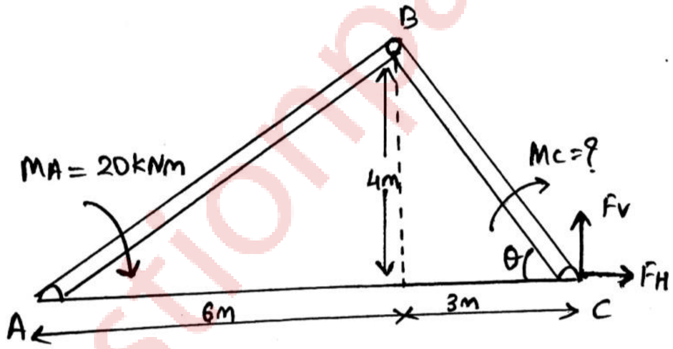
(b) Determine the moment to be applied at C for equilibrium of pin jointed mechanism. Use virtual work method. Refer Fig. (06 marks)



Solution:

From line BD:

Active Forces	Coordinates	Virtual Displacement
F_H	3	$5\cos\theta$
F_V	0	0



For maintaining equilibrium,

By Principal of virtual work,

$$\sum V.W = 0$$

$$\therefore F_V(0) + F_H(5\cos\theta) - 20 = 0$$

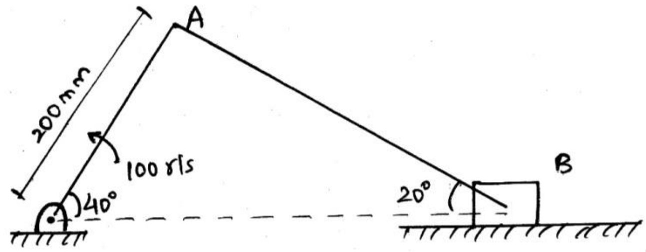
$$\therefore F_H = \frac{20}{5\cos\theta} = 4\sec\theta = \frac{20}{3} \text{ kN}$$

Hence the moment to be applied at point C is

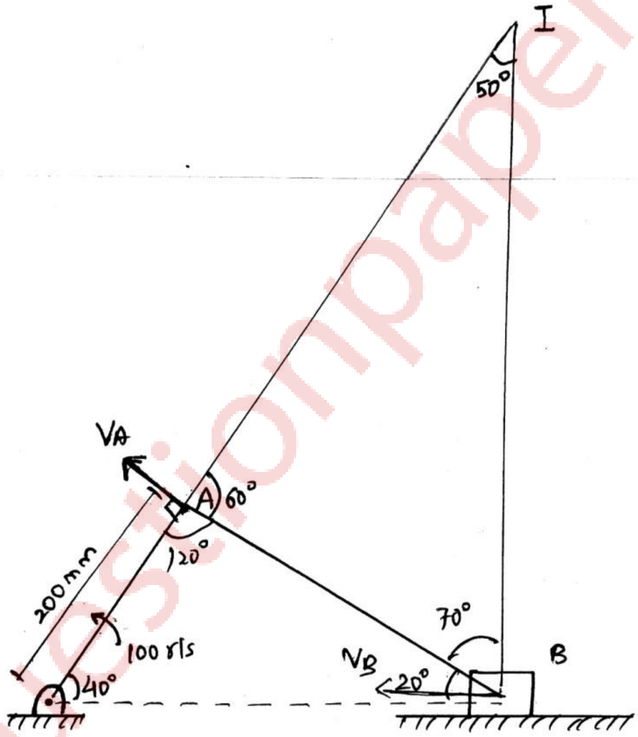
$$\boxed{M = \frac{20}{3}(3) = 20 \text{ kNm}}$$

(c) A slider crank mechanism is shown in Fig. The crank OA rotates anticlockwise at 100 rad/s. Find the angular velocity of rod AB and the velocity of the slider at B.

(06 marks)



Solution: Finding ICR,



By sine rule,

$$\frac{200}{\sin(20°)} = \frac{AB}{\sin(40°)} = \frac{OB}{\sin(120°)}$$

$\therefore$ AB=375.88 mm and OB=506.42 mm

By sine rule,

$$\frac{AB}{\sin(50°)} = \frac{IA}{\sin(70°)} = \frac{IB}{\sin(60°)}$$

$\therefore$ IA=461.09 mm and IB=424.94 mm

$$V_A = W_{OA}.OA$$

$$\therefore V_A = (100)\frac{200}{1000} = 20 \text{ m/s}$$

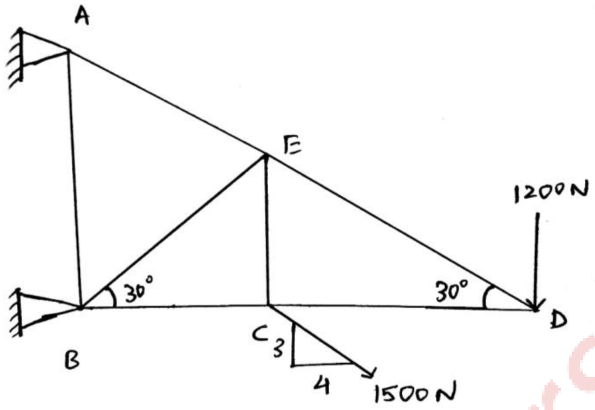
$$W_I = \frac{V_A}{IA} = \frac{20}{0.46} = 43.48 \text{ rad/s}$$

$$V_B = W_I \times IB = 43.48(0.42) = 18.26 \text{ m/s}$$

$$\boxed{\begin{array}{l} W_I = 43.48 \text{ rad/s} \\ V_B = 18.26 \text{ m/s} \end{array}}$$

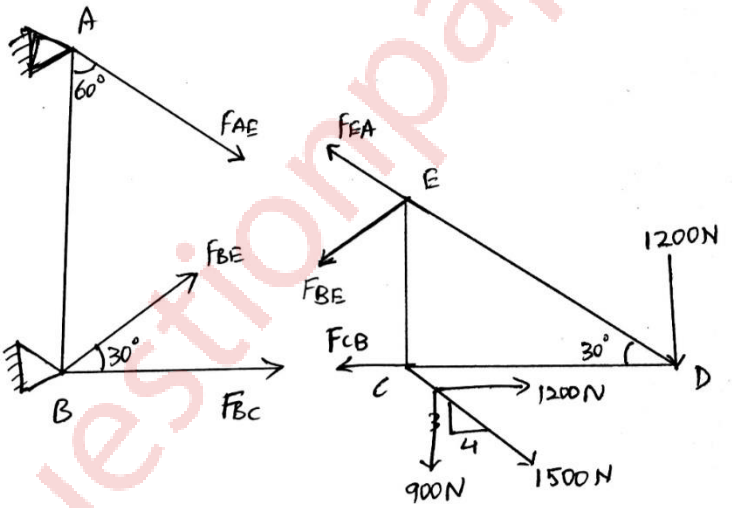
Q5. Attempt:

(a) Find the forces in the members BC, BE and AE by method of sections and remaining members by method of joints. (08 marks)



Solution:

By method of section, cutting the given truss along AE, BE and BC



Consider the right part,

$\sum F_X = 0$

$\therefore \quad -F_{EA}\cos(30°) - F_{BC} - F_{BE}\sin(60°) + 1200 = 0 \quad ....(1)$

$\sum F_Y = 0$

$\therefore \quad F_{EA}\sin(30°) - F_{BE}\cos(60°) - 1200 - 900 = 0 \quad ....(2)$

Let the length of hypotenuse be l

$\sum M_E^{\ F} = 0$

$-F_{BC}(l\sin(30°)) + 1200(l\sin(30°)) - 1200(l\cos(30°)) = 0$

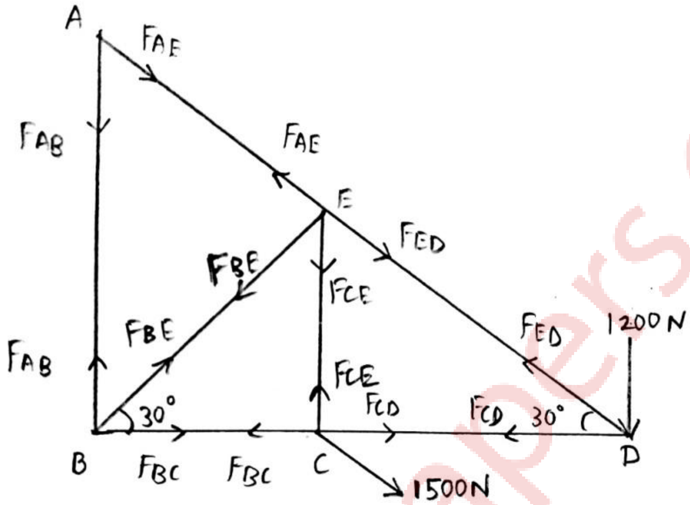
$\therefore \boxed{F_{BC} = -878.46\ N}$

$\therefore$ Put F_{BC} in (1)

$-F_{EA}\cos(30°)-(-878.46)-F_{BE}\sin(60°)+1200=0$

$\boxed{\begin{array}{l} F_{EA}=3300\text{ N} \\ F_{BE}=-900\text{ N} \end{array}}$

By method of joints,



At point D

$\sum F_Y=0$

$\therefore -1200+F_{ED}\sin(30°)=0$

$\therefore \boxed{F_{ED}=2400\text{ N}}$

$\sum F_X=0$

$\therefore -F_{CD}-F_{ED}\cos(30°)=0$

$\therefore \boxed{F_{CD}=-2078.46\text{ N}}$

At point C

$\sum F_X=0$

$\therefore F_{CD}-F_{BC}+1200=0$

$\therefore -2078.46-F_{BC}+1200=0$

$\therefore \boxed{F_{BC}=-878.46\text{ N}}$

$\sum F_Y=0$

$\therefore -900+F_{CE}=0$

$\therefore \boxed{F_{CE}=900\text{ N}}$

At point E

$\sum F_X = 0$

$\therefore -F_{AE}\cos(30°) - F_{BE}\sin(60°) + F_{ED}\cos(30°) = 0$

$\therefore -F_{AE}\cos(30°) - F_{BE}\sin(60°) + 2400\cos(30°) = 0$

$\sum F_Y = 0$

$\therefore F_{AE}\sin(30°) - F_{BE}\cos(60°) - F_{CE} - F_{ED}\sin(30°) = 0$

$\therefore F_{AE}\sin(30°) - F_{BE}\cos(60°) - 900 - 2400\sin(30°) = 0$

$$\boxed{\begin{aligned} F_{AE} &= 3300\text{ N} \\ F_{BE} &= 900\text{ N} \end{aligned}}$$

At point A

$\sum F_Y = 0$

$\therefore -F_{AB} - F_{AE}\cos(60°) = 0$

$\therefore F_{AB} = -3300\cos(60°)$

$\therefore \boxed{F_{AB} = -1650\text{ N}}$

Member	Force Magnitude (in N)	Nature of Force
AB	1650	Compressive
BE	900	Tensile
BC	878.46	Compressive
AE	3300	Tensile
CE	900	Tensile
ED	2400	Tensile
CD	2078.46	Compressive

(b) A particle moves in x-y plane and it's is given by $r = (3t)i + (4t - 3t^2)j$, where r is the position vector of particle in metres at time t sec. Find the radius of curvature of the path and normal and tangential components of acceleration when it crosses X-axis region. (06 marks)

Solution:

$\bar{r} = \left[(3t)i + (4t - 3t^2)j\right]$ m

When it crosses the x axis, the y coordinate is 0

$\therefore (4t - 3t^2) = 0$

$\therefore t = 0s$ or $t = \frac{4}{3}s$

$\bar{v} = \frac{d\bar{r}}{dt} = \frac{d}{dt}\left[(3t)i + (4t - 3t^2)j\right]$

$\therefore \bar{v} = \left[3i + (4 - 6t)j\right]$ m/s

$\therefore$ At t=$\frac{4}{3}s$,

$\bar{v} = \left[3i - 4j\right]$ m/s=$5\angle -53.13°$ m/s

$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}\left[3i + (4 - 6t)j\right]$ m/s^2

$\therefore \bar{a} = -6j$ m/s^2

Radius of curvature(ρ)=$\frac{v^3}{\left|v_x a_y - v_y a_x\right|}$ m

$\therefore \rho = \frac{5^3}{\left|(3)(-6) - (-4)(0)\right|} = \frac{125}{\left|-18\right|} = 6.94$ m

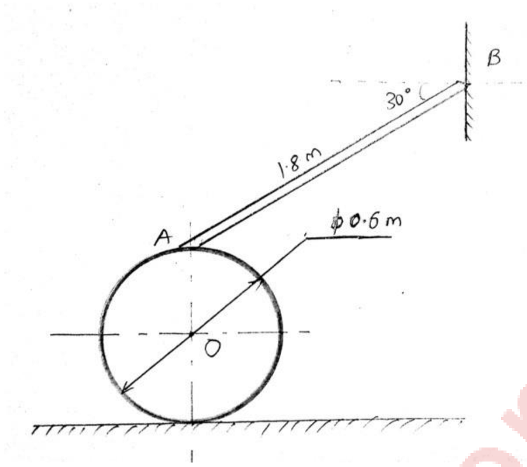
$a_N = \frac{v^2}{\rho} = \frac{5^2}{6.94} = \frac{25}{6.94} = 3.6$ m/s^2

$a_T = \sqrt{a^2 - a_N{}^2} = \sqrt{6^2 - 3.6^2} = 4.8$ m/s^2

$\therefore$

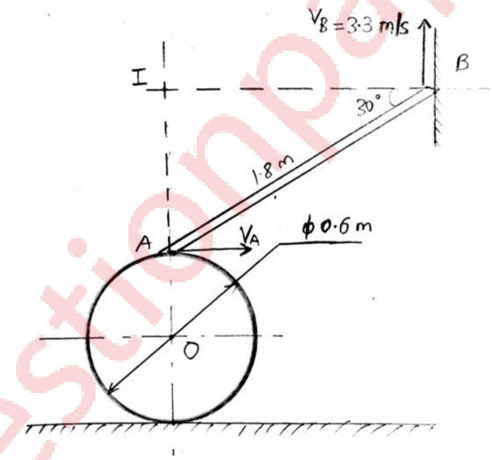
Radius of curvature=6.94 m and
Normal component of acceleration=3.6 m/s^2 and
Tangential component of acceleration=4.8 m/s^2

(c) C is a uniform cylinder to which a rod AB is pinned at A and the other end of the rod is moving along a vertical wall as shown in fig. If the end B of the rod is moving upwards along the wall with a speed of 3.3 m/s find the angular velocity of wheel and rod assuming that cylinder is rolling without slipping. (06 marks)



Solution:

Locating the ICR,



$$IB=1.8\cos(30°)=1.56\text{ m and }IA=1.8\sin(30°)=0.9\text{ m}$$

$$V_B = 3.3\text{ m/s}$$

$$W_I = \frac{V_B}{IB} = \frac{3.3}{1.56} = 2.12\text{ r/s}$$

$$V_A = W_I \times (IA)=(2.12)(0.9)$$

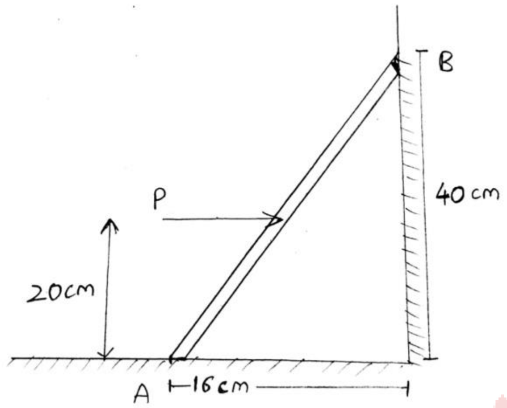
$$\therefore V_A = 1.91\text{ m/s}$$

$$\therefore W=\frac{V_A}{R} = \frac{1.91}{0.3} = 6.37\text{ r/s}$$

∴ The angular velocity of wheel =6.37 r/s and the velocity of the rod =1.91 m/s

Q6.Attempt:

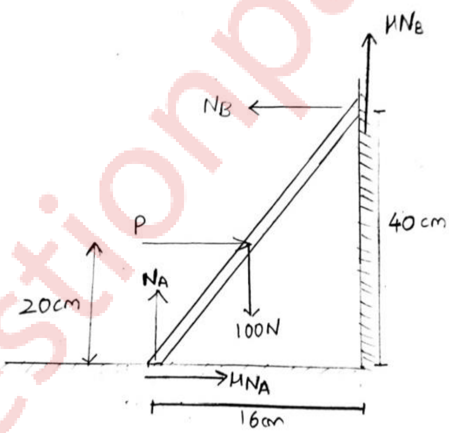
(a) A 100 N uniform rod AB is held in position as shown. If $\mu = 0.15$ at A and B calculate range of value of P for which equilibrium is maintained. (08 marks)



Solution:

Let N_A and N_B be the normal reactions at A and B respectively

For minimum value of P, FBD is,



For equilibrium,

$\sum F_x = 0$

$\therefore \mu N_A - N_B + P = 0$

$\therefore 0.15N_A - N_B + P = 0 \quad ...(1)$

$\sum F_y = 0$

$\therefore N_A + \mu N_B - 100 = 0$

$\therefore N_A + 0.15N_B + 0P = 100 \quad ...(2)$

$\sum M_A = 0$

$\therefore P\left(\frac{20}{100}\right) + 100\left(\frac{8}{100}\right) - N_B\left(\frac{40}{100}\right) - \mu N_B\left(\frac{16}{100}\right) = 0$

$\therefore 0N_A - 42.4N_B + 20P = -800 \qquad ...(3)$

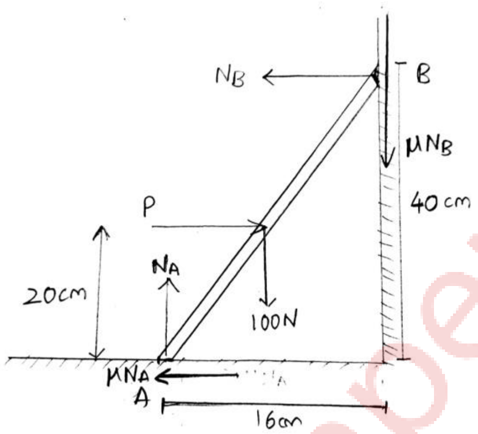
From (1), (2) and (3),

$N_A = 96.58$ N

$N_A = 22.78$ N

P=8.29 N

For maximum value of P, FBD is,



For equilibrium,

$\sum F_x = 0$

$\therefore -\mu N_A - N_B + P = 0$

$\therefore -0.15N_A - N_B + P = 0 \qquad ...(4)$

$\sum F_y = 0$

$\therefore N_A - \mu N_B - 100 = 0$

$\therefore N_A - 0.15N_B + 0P = 100 \qquad ...(5)$

$\sum M_A = 0$

$\therefore P\left(\frac{20}{100}\right) + 100\left(\frac{8}{100}\right) - N_B\left(\frac{40}{100}\right) + \mu N_B\left(\frac{16}{100}\right) = 0$

$\therefore 0N_A - 37.6N_B + 20P = -800 \qquad ...(6)$

From (4), (5) and (6),

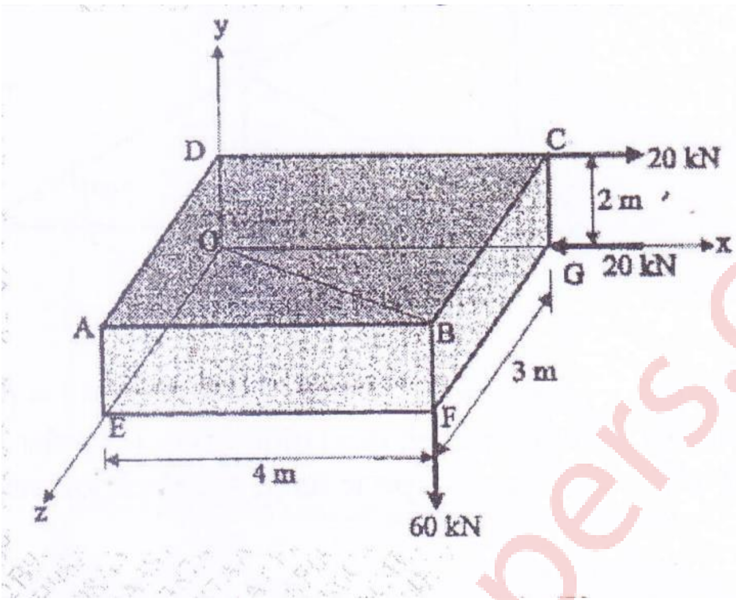
$N_A = 109.62$ N

$N_A = 64.14$ N

P=80.58 N

$\therefore$ The range of value of P for which equilibrium is maintained is from 8.29 N to 80.58 N

(b) A box of size $3 \times 4 \times 2$ m is subjected to three forces as shown in fig. Find in vector form the sum of moments of the three forces about diagonal OB. (06 marks)



Solution:

The three forces are given as

$$\overline{F_{DC}} = 20\hat{i} \text{ kN}$$
$$\overline{F_{GO}} = -20\hat{i} \text{ kN}$$
$$\overline{F_{BF}} = -60\hat{j} \text{ kN}$$

The unit vector along the direction OB is

$$\hat{OB} = \frac{(4-0)\hat{i} + (2-0)\hat{j} + (3-0)\hat{k}}{\sqrt{4^2 + 2^2 + 3^2}}$$
$$= \frac{4\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{29}}$$

The vector moment of force F_{DC} along OB is

$$\overline{M} = \left[\overline{OC} \quad F_{DC} \quad \hat{OB} \right] \hat{OB}$$

$$\overline{M} = \begin{vmatrix} 4 & 2 & 0 \\ 20 & 0 & 0 \\ \frac{4}{\sqrt{29}} & \frac{2}{\sqrt{29}} & \frac{3}{\sqrt{29}} \end{vmatrix} \hat{OB}$$

$$\therefore \overline{M} = -20\left(\frac{6}{\sqrt{29}}\right)\hat{OB}$$

$$\overline{M} = \frac{-120}{\sqrt{29}}\left[\frac{4\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{29}}\right]$$

$$\therefore \boxed{\overline{M} = -16.55\hat{i} - 8.28\hat{j} - 12.41\hat{k}}$$

The vector moment of force F_{GO} along OB is

$$\overline{M} = \left[\overline{OG} \quad F_{GO} \quad \hat{OB}\right]\hat{OB}$$

$$\overline{M} = \begin{vmatrix} 4 & 0 & 0 \\ -20 & 0 & 0 \\ \frac{4}{\sqrt{29}} & \frac{2}{\sqrt{29}} & \frac{3}{\sqrt{29}} \end{vmatrix}\hat{OB}$$

$$\therefore \overline{M} = 20\left(\frac{6}{\sqrt{29}}\right)\hat{OB}$$

$$\overline{M} = \frac{120}{\sqrt{29}}\left[\frac{4\hat{i}+2\hat{j}+3\hat{k}}{\sqrt{29}}\right]$$

$$\therefore \boxed{\overline{M} = 16.55\hat{i} + 8.28\hat{j} + 12.41\hat{k}}$$

The vector moment of force F_{BF} along OB is

$$\overline{M} = \left[\overline{OB} \quad F_{BF} \quad \hat{OB}\right]\hat{OB}$$

$$\overline{M} = \begin{vmatrix} 4 & 2 & 3 \\ -20 & 0 & 0 \\ \frac{4}{\sqrt{29}} & \frac{2}{\sqrt{29}} & \frac{3}{\sqrt{29}} \end{vmatrix}\hat{OB}$$

$\therefore \overline{M} = 0\hat{OB}$ ....{Since 2 rows of the matrix are equal}

$$\therefore \boxed{\overline{M} = 0\hat{i} + 0\hat{j} + 0\hat{k}}$$

(c) Two blocks A and B are separated by 10 m as shown in fig on $20°$ incline plane. If the blocks start moving, find the time t when the blocks collide and distance travelled by each block. Assume $\mu_k = 0.3$ for block A and block A and plane and $\mu_k = 0.10$ for block B and plane. **(06 marks)**

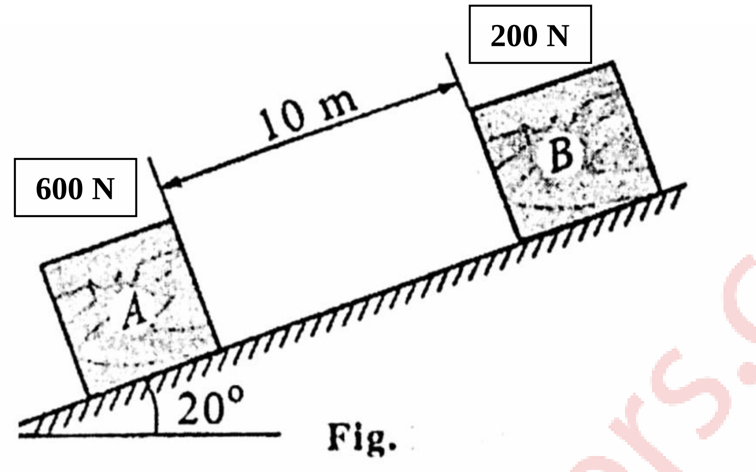
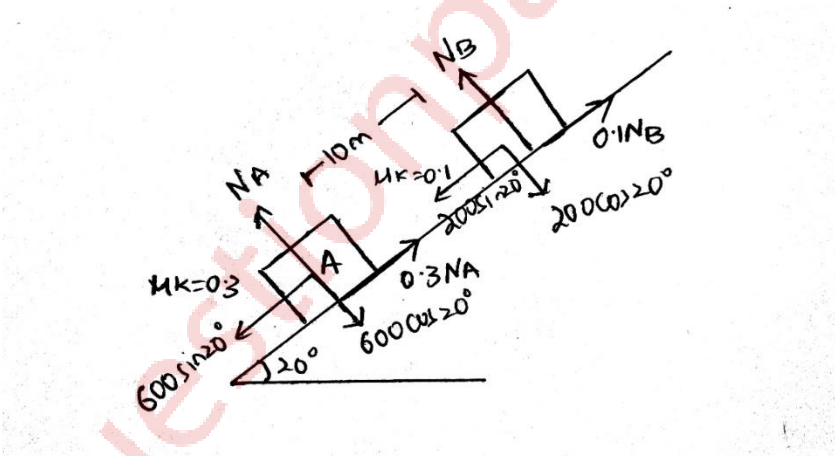


Fig.

Solution:

The FBD is:



Let the time taken by the blocks to meet be t seconds

Let the distance travelled by block A be x m

$\therefore x = 0 + \frac{1}{2}(a_A)t^2$ .....(1), a_A = Acceleration of block A

Hence, distance travelled by block B is

$x + 10 = 0 + \frac{1}{2}(a_B)t^2$ .....(2), a_B = Acceleration of block B

From the FBD,

On block A,

$$\text{mass of block A}=m_A=\frac{600}{g}=\frac{600}{9.8}=61.22\text{ kg}$$

$$600\cos(20°)=(\mathrm{N_A})$$

$$\therefore \mathrm{N_A}=563.82\text{ N}$$

$$600\sin(20°)-0.3\mathrm{N_A}=m_\mathrm{A}a_A$$

$$\therefore\ a_A=0.589\text{ m/s}^2$$

On block B,

$$\text{mass of block B}=m_B=\frac{200}{g}=\frac{200}{9.8}=20.41\text{ kg}$$

$$200\cos(20°)=(\mathrm{N_B})$$

$$\therefore \mathrm{N_B}=187.94\text{ N}$$

$$200\sin(20°)-0.1\mathrm{N_B}=m_\mathrm{B}a_B$$

$$\therefore\ a_B=2.43\text{ m/s}^2$$

By putting the values of a_A and a_B in equations (1) and (2),

$$x=\frac{1}{2}(0.589)t^2\text{ .....(3)} \qquad \text{and} \qquad x+10=\frac{1}{2}(2.43)t^2 \qquad \text{.....(4)}$$

Dividing equation (3) by (4),

$$\frac{x}{x+10}=\frac{\frac{1}{2}(0.589)t^2}{\frac{1}{2}(2.43)t^2}$$

$$\therefore x=3.2\text{ m}$$

From (3)

$$\text{t}=\sqrt{\frac{2(3.2)}{0.589}}=3.3\text{ s}$$

The blocks collide after time=3.3 seconds and the distance travelled by block A is 3.2 m and that by block B is (3.2+10) m=13.2 m.

Q.P. code :- 58653

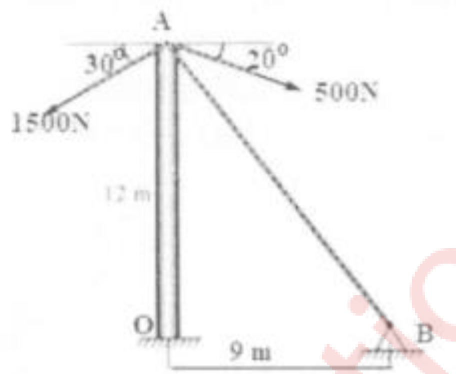
ENGINEERING MECHANICS – SEMESTER – 1

CBCGS DEC 2019

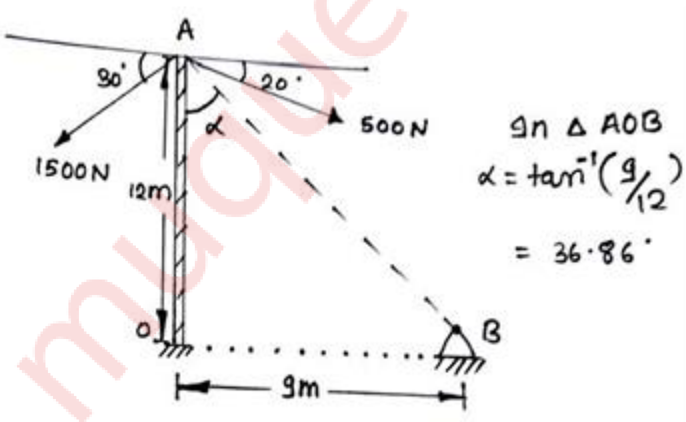
1.)

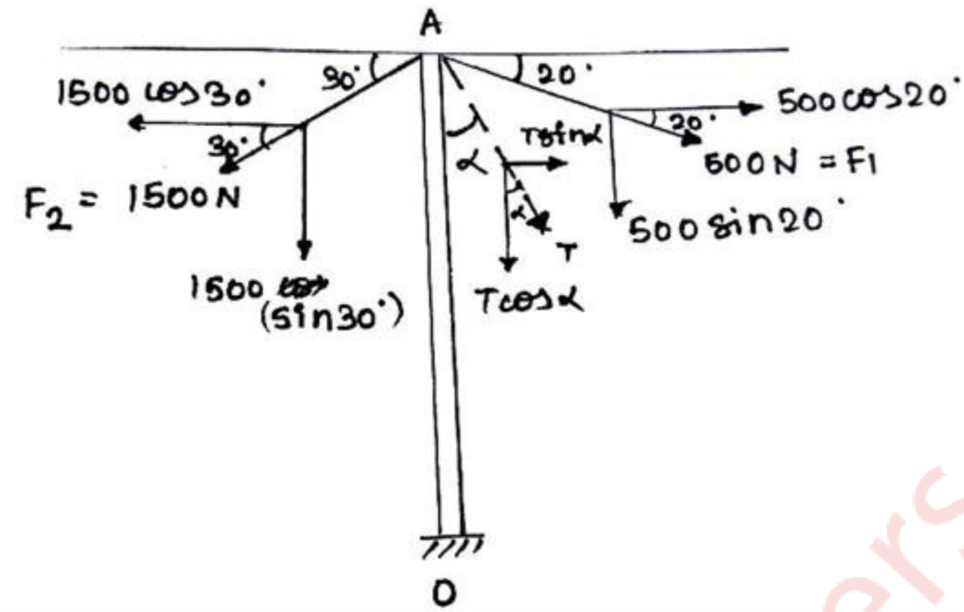
a. **(4)**

The top end of a pole is connected by three cables having. tension 500 N ,1500 N and a guy wire 'AB' as shown in figure below. Determine tension in cable 'AB' if the resultant of the concurrent forces is vertical.



Soln:-





Given:- In the figure , F1=500N,F2=1500N,T=?,F1 makes an angle 20° with horizontal and F2 makes 30° with the horizontal and assume tension 'T' makes an angle α with vertical.

Resultant force in vertical direction.

To find:- Tension 'T'=?

CALCULATION :-

IN ΔAOB

$\alpha = 36.86°$

Taking forces having direction towards right as positive and forces having direction upwards as

Positive.

Resolving forces along X direction :

$Rx = F\ 1 \cos20° – F\ 2 \cos30° + T\sin\alpha$

$= 500\cos20° – 1500\cos30° + T\sin\alpha$ ....(1)

Resolving forces along Y direction:

$Ry = -F\ 1 \sin20° - F\ 2 \sin30° - T \cos\alpha$

= -500sin20° - 1500sin30° - Tcosα

..........(2)

Resultant force is in upward direction so

Rx=0

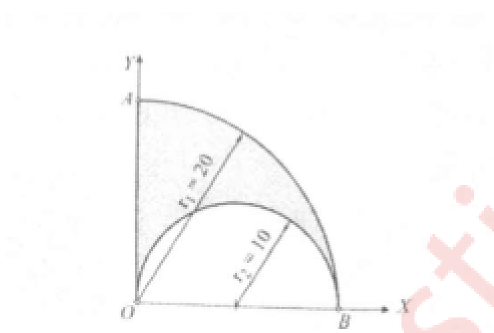
Put Rx = 0 ,α=36.86°(calculated) in equation 1

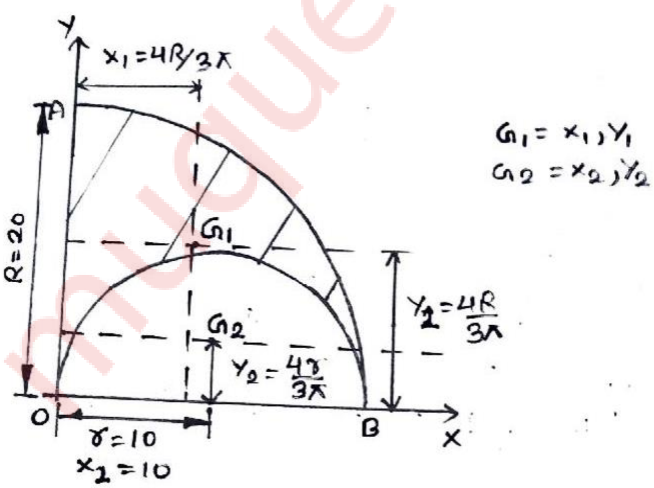
500cos20° - 1500cos30° + Tsin36.86° = 0

T = 1382.304N (ANS)

b. **(4)**

Locate the centroid of the shaded area obtained by cutting a semicircle of diameter 20mm from quadrant of a circle of radius 20mm as shown in figure below.





Soln:-

Given:- Co-ordinates of shaded portion can be obtained by taking a quater circle of radius 20mm and subtracting a semi-circle of radius 10mm.

To find:- centroid co-ordinates.

Calculation:-

PART	Area (Ai) mm^2	Xi mm	Yi mm	Aixi mm^3	Aiyi mm^3
1.Quater circle	**314.15**	**8.488**	**8.488**	**2666.50**	**2666.50**
2.semi-circle	**-157.08**	**10**	**4.244**	**-1570.8**	**-666.64**

X co-ordinate of centroid ($\bar{x}$) = ΣAxi/ΣA = 1095.7/157.07 = 6.97mm

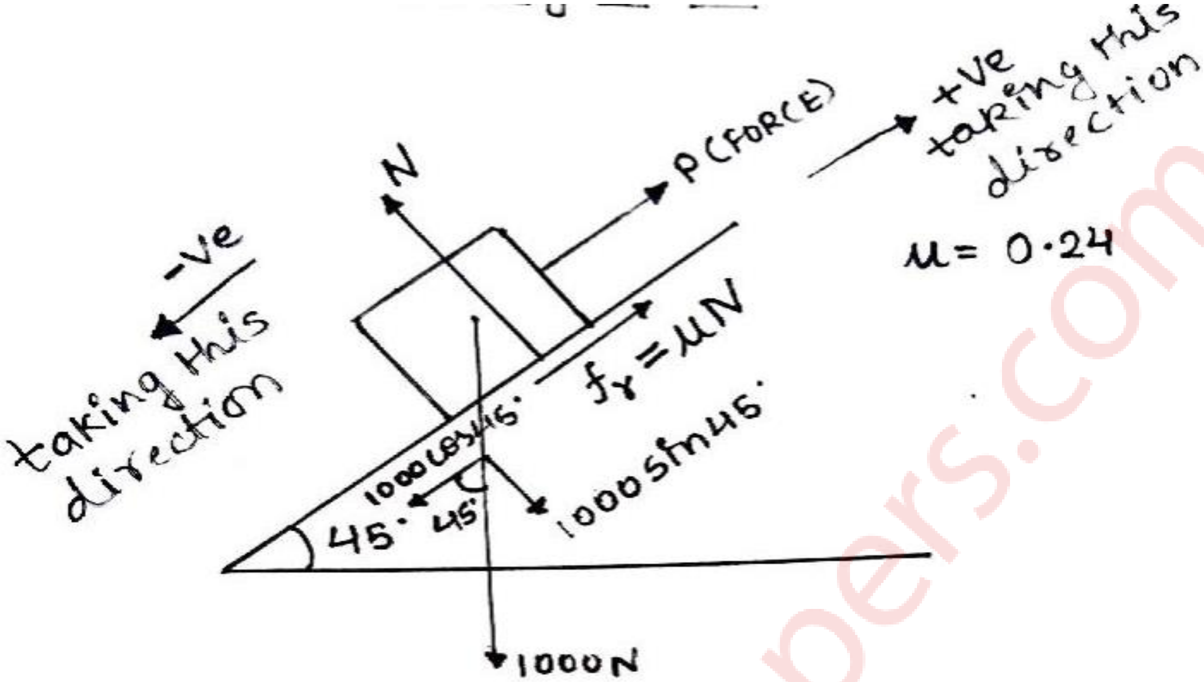
Y co-ordinate of centroid($\bar{y}$) =ΣAyi/ΣA = 1999.86/157.07 = 12.73mm

Centroid is at (6.97,12.73)mm (ANS)

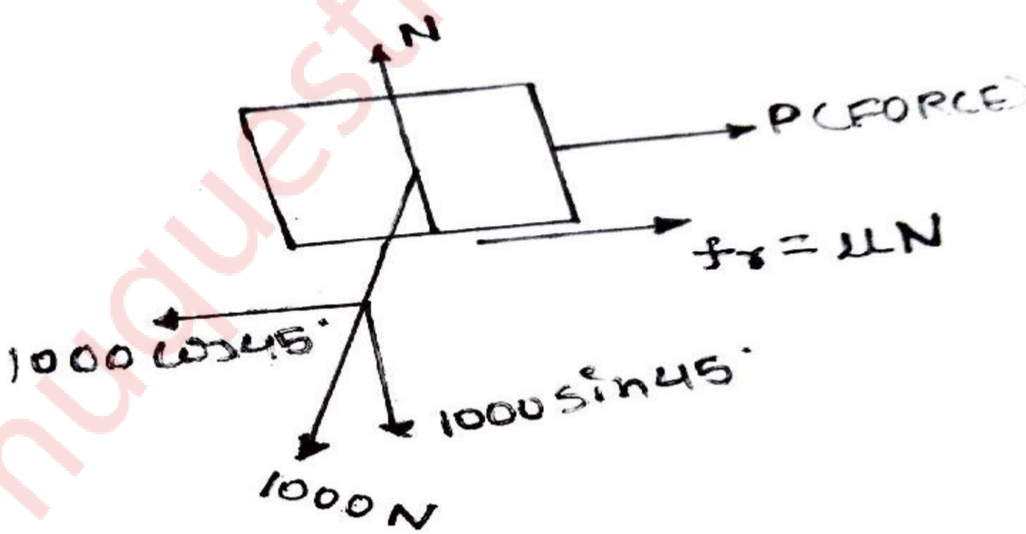
c. **(4)**

A body weighing 1000N is lying on a horizontal plane. Determine necessary force to move the body along the plane if the force is applied at an angle of 45 degrees to the horizontal with coefficient of friction 0.24.

Soln:-



FBD OF BLOCK :-



Let the normal force be 'N' and friction force be 'fr' and force 'P' be the force required to keep the body in equilibrium and +ve as X-axis and –ve as Y-axis .

Applying equilibrium conditions on Block,

$\Sigma F_X = 0$

P + fr – 1000cos(45°) = 0 ....(1)

$\Sigma F_Y = 0$

N – 1000sin(45°) = 0 ....(2)

N = 707.106 N (put this in equation (1))

P + μN - 1000cos(45°) = 0 (fr=μN,μ=0.24)

P = 1000cos(45°) - 0.24*(707.106)

The minimum weight of P is

P = 537.40N (ANS)

d. **(4)**

The motion of the particle is defined by the relation x = t^3-3t^2+2t+5 where x is the position expressed in meters and time in seconds. Determine (i) the velocity and acceleration after 5 seconds. (ii)maximum or minimum velocity and corresponding displacement.

Soln:-

Given :- Rightward as +ve and Leftward as –ve.

x(t)= t^3-3t^2+2t+5

v(t)= dx/dt

= 3t^2-6t+2

a(t)= dv/dt

= 6t-6

(i) v(5) = 3(5)^2-6(5)+2

= 47m/s^2

a(5) = 6(5)-6

= 24m/s^2

(ii) maximum or minimum velocity and corresponding displacement,happens only when dv/dt = 0 . Put dv/dt = 0

6t-6=0

t=1

If d2v*/dt*2 is positive then their will be minima otherwise maxima.

d2v/dt2 = 6 (positive)

So, minimum velocity exist .

At t=0,

v(1)= 3(1)^2-6(1)+2

= -1 or 1m/s^2 (in left direction)

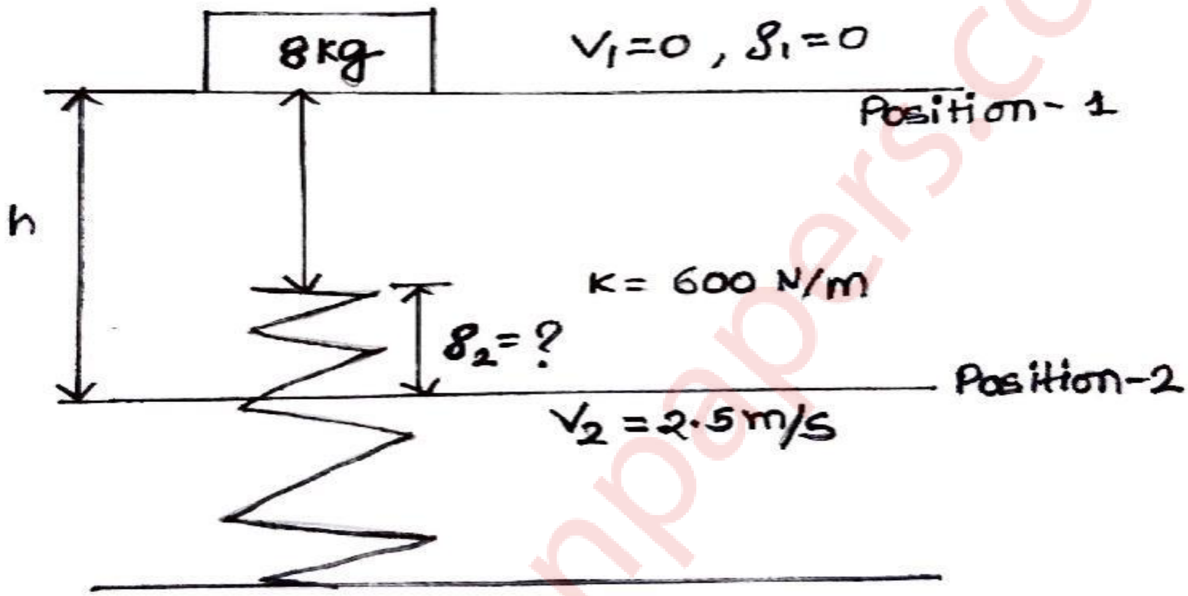
x(1)=(1)^3-3(1)^2+2(1)+5

= 5m **(ANS)**

e. **(4)**

A steel ball of mass 8 kg is dropped onto a spring of stiffness 600 N/m and attains a maximum velocity 2.5 m/s. Find (i) the height from it is dropped and (ii) the maximum deflection of spring.

Soln:-



Given:- The fall is free and the starts with zero velocity ,

At position 1 ,the velocity is zero , delection is zero ,K= 600N/m , mass of body = 8Kg ,

At position 2 , the velocity is 2.5m/s^2 and deflection say 'δ',

Work done :-

(i work done by weight = mgh

= 8*9.81*h (v^2=u^2+2g(h- δ2)

(this equation is applied for free fall body so height will be h- δ2 because after that motion is influenced by spring.)

$((2.5)^2=0+2*9.81*(h-\delta 2))$

$(h=(0.31855+\delta 2)\ m)$

$= 8*9.81*(0.31855+\delta 2)$

$=(25+78.48\delta 2)\ J$

(ii work done by spring = $\frac{1}{2}*k*((\delta 1)^2 - (\delta 2)^2)$

$= \frac{1}{2}*(600)*(0-(\delta 2)^2)$

$= -300(\delta 2)^2\ J$

summation of all work done = $\Sigma U_{1\text{-}2}$

$= (25+78.48\delta 2) - 300(\delta 2)^2$

BY WORK ENERGY THEOREM:-

$T1 + \Sigma U_{1\text{-}2} = T2$

T1= INITIAL KINETIC ENERGY = $\frac{1}{2}*m*v1^2 = \frac{1}{2}*8*0$

T2 = FINAL KINETIC ENERGY = $\frac{1}{2}*m*v2^2 = \frac{1}{2}*8*(2.5)^2 = 25$

$0 + (25+78.48\delta 2) - 300(\delta 2)^2 = 25$

$(\delta 2) = 0.2616$ or 0

$(\delta 2) = 0.2616$ m (maximum deflection)(ii ans)

$h= (\delta 2) + 0.31855$

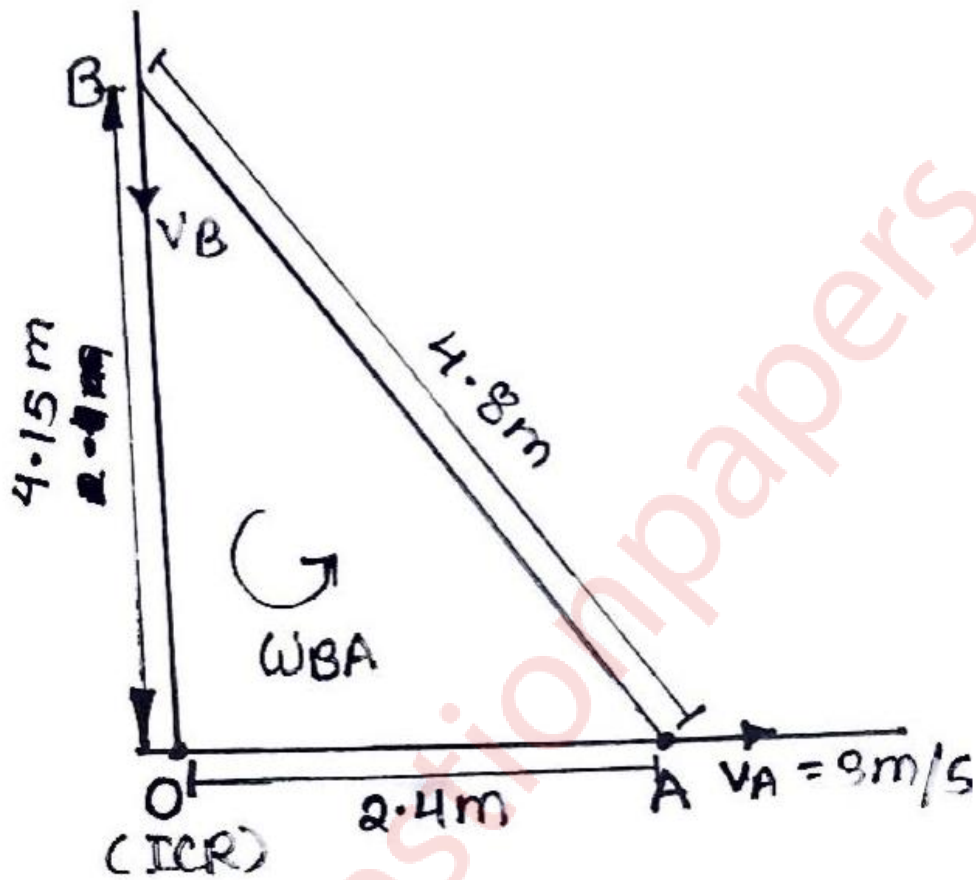
$= 0.58015$ m (height from where it is dropped)(i ans)

f. **(4)**

A ladder AB of length l=4.8 m rests on a horizontal floor at A and leans against a vertical wall at B. If the lower end A is pulled away

from the wall with a constant velocity 3 m/s, what is the angular velocity of the ladder at the instant when A is 2.4 m from wall.

Soln:-



Given :- In **ΔAOB ,** OA and AB is given in the problem. Velocity of A is 3m/s and at b is unknown. Point of rotation or instantaneous center of rotation is O. Rotation is from B to A.

To find:- ω_{BA} = ?

Calculation:- In **ΔAOB,** OA and AB are 2.4m and 4.8m resp., By pythagoras theorem ,

OB = $\sqrt{AB^2 - OA^2}$

$= \sqrt{4.8^2 - 2.4^2}$

$= 4.15m$

Instantaneous center of rotation is the point of intersection of

vA and vB velocity vector.

Radius of rotation is perpendicular distance of velocity vector of point from instantaneous center of rotation (ICR).

So, $r_A = 2.4m$

$r_B = 4.15m$

Instantaneous velocity of $v_A = \omega_{BA} * r_A$.

$3 = \omega_{BA} * (2.4)$

Angular Velocity of motion from B to A = ω_{BA} = 1.25 rad/sec (ANS)

Instantaneous velocity of $v_B = \omega_{BA} * r_B$

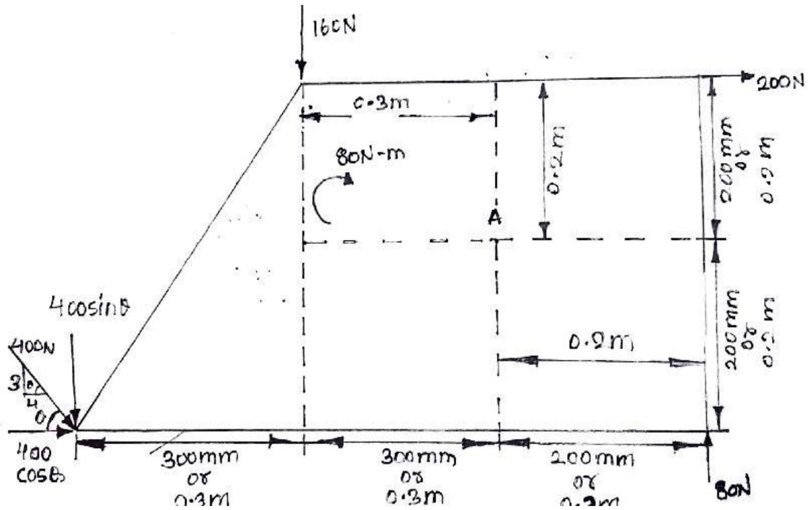
$v_B = 1.25 * 4.15$

$= 5.18m/s$

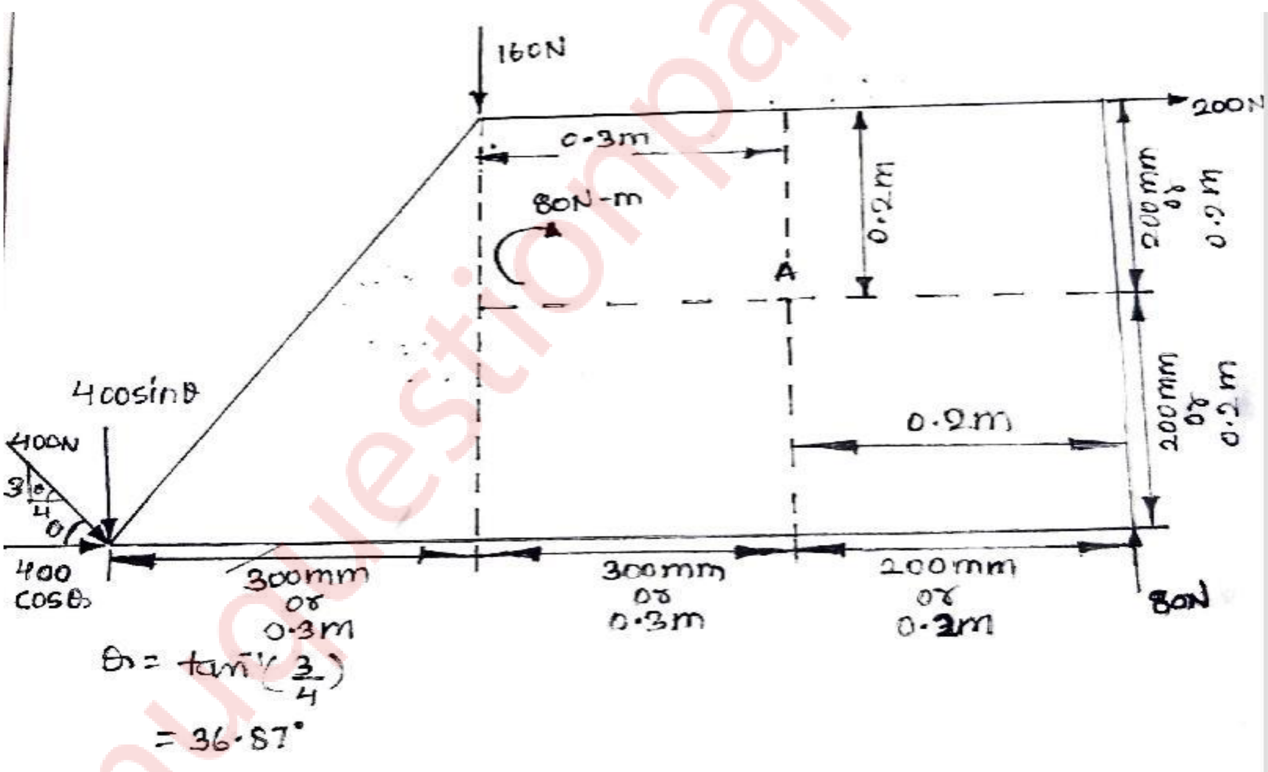
2.)

a. **(8)**

Find the resultant of the force system acting on the plate as shown in figure, where does this resultant act with respect to point A ?



Soln:-



Given:- The figure 1st and 2nd .

To find:- R**esultant act with respect to point A**

Calculation:-

$\Sigma Fx = 200 + 400\cos(36.87°)$ (taking right as +ve)

$= +519.99$ N or 519.99 N (rightwards)

$\Sigma Fy = 80 – 160 – 400\sin(36.87°)$ (taking upwards as +ve)

$= -320.01$ N or 320.01 N (downwards)

$R = \sqrt{(Fx)^2 + (Fy)^2}$

$= \sqrt{(519.99)^2 + (320.01)^2}$

$= 610.57$ N

$\Theta = \tan^{-1}(Fy/Fx)$

$= \tan^{-1}(320.01/519.99)$

$= 31.60°$

MOMENT OF ALL THE FORCES ABOUT POINT 'A'.

MOMENT = F*(Perpendicular distance of line of force from the point)

$\Sigma M_A = -200*(0.2) + 80*(0.2) + 160*(0.3) - 400\sin(36.87°)*(0.6)$

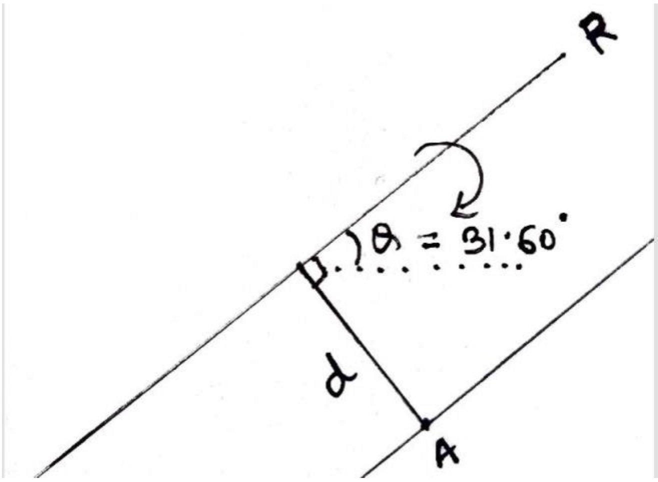
(taking anti-clockwise as +ve)

$= -120$N-m or 120N-m(clockwise)

$\Sigma M_A = R*d$

$120 = 610.57*d$

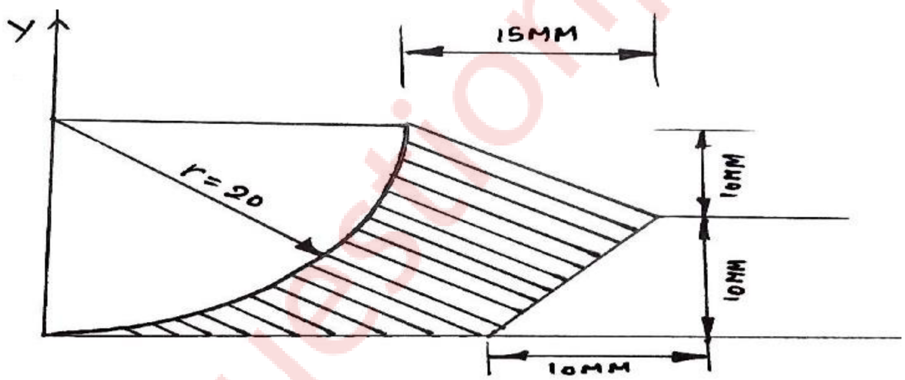
$d = 0.1965$ m



THE RESULTANT OF FORCE IDS ACTING CLOCKWISE ABOUT POINT 'A' AND AT A DISTANCE OF d = 0.196m or 196mm. (ans)

b. **(6)**

Find centroid of the shaded area with reference to X and Y Axes.



Soln:-

Given:- G1,G2,G3,G4 are centroids of respective figures.

Area of the shaded region = Rectangle ACHE - Quarter Circle ABC – Triangle BHF - Triangle DEF

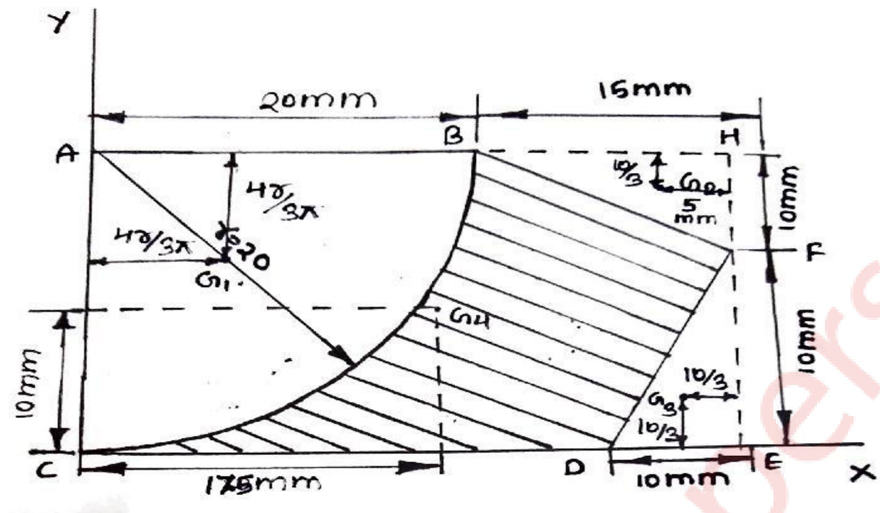


Figure	AREA (mm^2)	X-coordinate (mm)	Y-coordinate (mm)	Aixi (mm^3)	Aiyi (mm^3)
Rectangle ACHE	35*20 = 700	17.5	10	12250	7000
Quarter circle ABC	- ¼*Π*r^2 = -314.15	4r/3Π = 4*20/3Π =8.48	20- 4r/3Π = 20- 4*20/3Π = 11.52	-2663.99	-3619.008
Triangle BHF	- ½*10*15 =-75	35-5=30	20-10/3 =16.67	-2250	-1250.25
Triangle DEF	- ½*10*10 =-50	35-10/3 = 31.67	10/3 = 3.33	-1583.5	-166.5

ΣAi = 700 – 314.15 - 75 – 50 = 260.85 mm^2

ΣAixi = 12250 – 2663.99 - 2250 – 1583.5 = 5752.51 mm^3

ΣAiyi = 7000 – 3619.008 - 1250.25 - 166.5 = 1964.242 mm^3

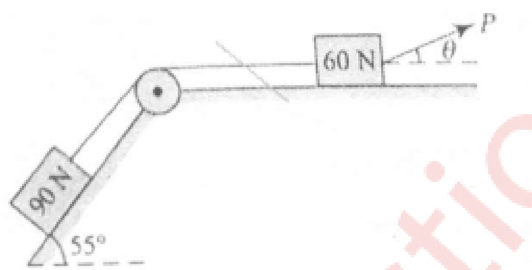
$\bar{x}$ = ∑Aixi/∑Ai = 5752.51/260.85 = 22.05mm

$\bar{y}$ = ∑Aiyi/∑Ai = 1964.242/260.85 = 7.53mm

Centroid coordinates of lamina is (22.05mm, 7.53mm) (ANS)

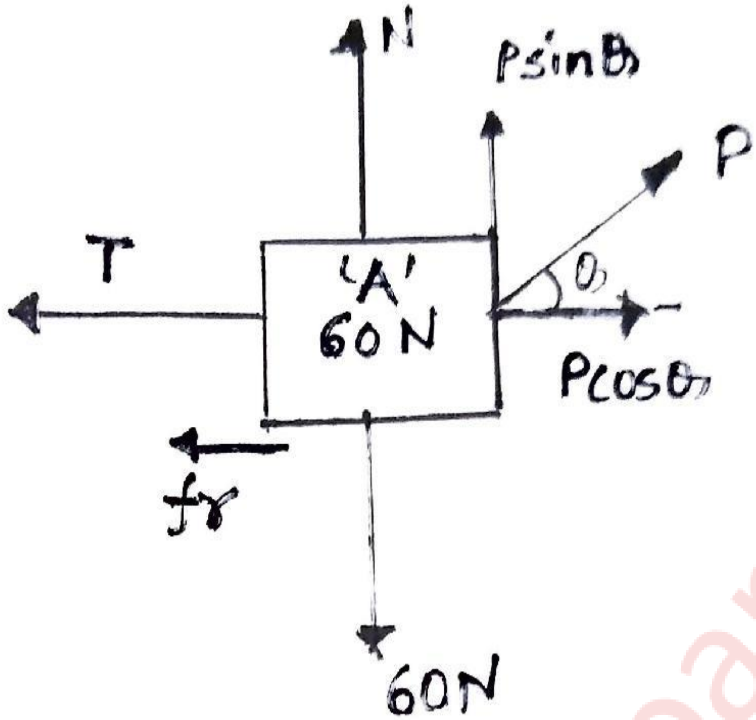
c. **(6)**

Two bodies A and B weighing 90 N and 60 N respectively placed on an inclined plane are connected by the string which is parallel to the plane as shown in Fig. Find the inclination of the minimum force P for the motion to impending the direction of "p". Take μ = 0.2 for the surface of contact.



Soln:-

FBD of 60N block:-



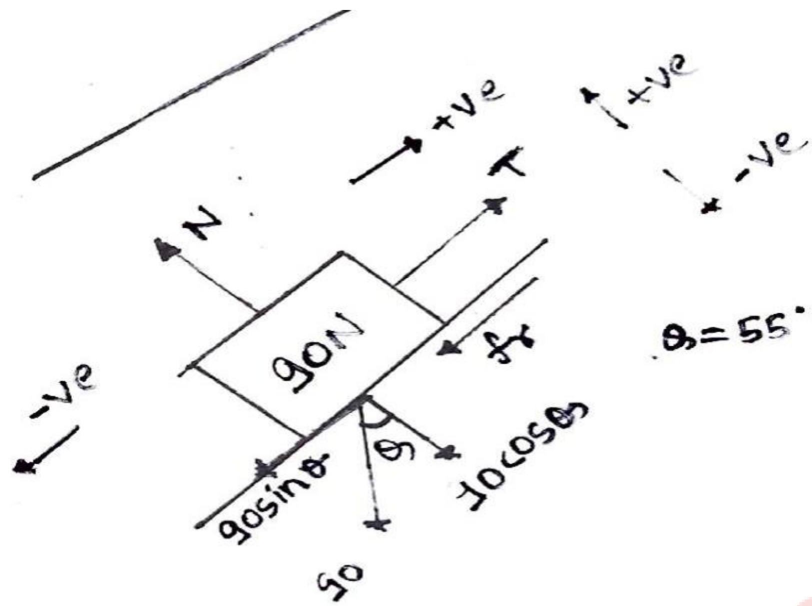
Consider block A

∑Fx=0 (consider right as +ve)

Pcosθ - T - fr= 0 ....(1)

∑Fy=0 (Consider upward as +ve)

Psinθ + N_A – 60 = 0 ......(2)



Consider block 90N

∑Fx=0 (consider right as +ve)

T – 90sin(55°) - fr = 0 ....(3) (fr = μ*N_B, μ= 0.2)

∑Fy=0 (Consider upward as +ve)

N_B – 90cos(55°) = 0 ......(4)

N_B = 51.62 N (put in equation 3)

T = 90sin(55°) + μ*N_B

= 84.04 N (put in equation 1)

Pcosθ - T - fr= 0 (fr = μ*N_A, μ= 0.2)

Pcosθ - 84.04 - 0.2*N_A = 0

Psinθ + N_A – 60 = 0 (equation 2)

N_A = 60 – Psinθ

Put in equation 1

$P\cos\theta - 84.04 - 0.2*(60 - P\sin\theta) = 0$

$P\cos\theta - 84.04 - 12 + 0.2* P\sin\theta = 0$

$P\cos\theta + 0.2* P\sin\theta = 96.04$

$P = 96.04/(\cos\theta + 0.2*\sin\theta)$

To minimize P, differentiate then equate to zero

$dP/d\theta = -96.04*(-\sin\theta + 0.20\cos\theta)/(\cos\theta + 0.20\sin\theta)^2 = 0$

$-\sin\theta + 0.20\cos\theta = 0$

$\sin\theta = 0.20\cos\theta$

$\tan\theta = 0.20$

$\theta = 11.31^\circ$

Thus,

$P_{min} = 96.04/(\cos 11.31^\circ + 0.20\sin 11.31^\circ)$

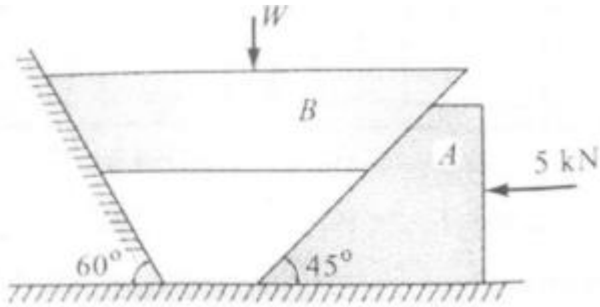
$P_{min} = 94.174$ kN

(*answer)*

3.)

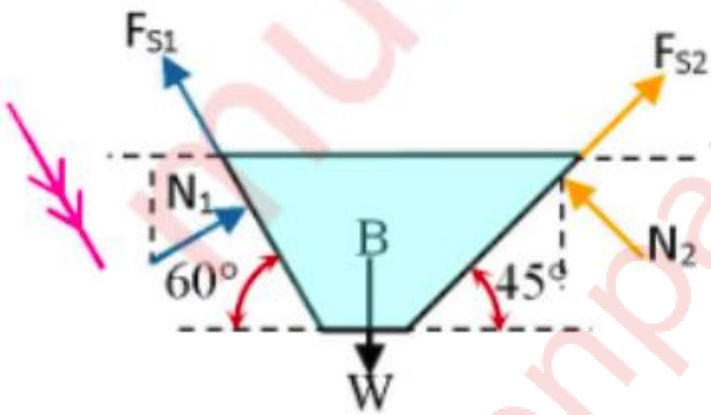
a. **(8)**

A horizontal force of 5KN is acting on the wedgeas shown in figure. The coefficient of friction at all rubbing surfaces is 0.25. Find the load "W" which can be held in position. The weight of block "B" may be neglected.



Soln:-

Let N 1 , N 2 , N 3 ,be the normal reaction at the surface of contact



∴ F S1 = μ 1 N 1 = 0.25N 1 , F S2 = μ 2 N 2 = 0.25N 2 , F S3 = μ 3 N 3 = 0.25N 3 ....(1)

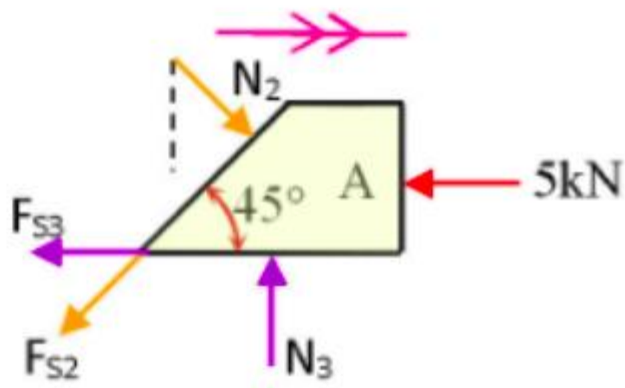
Block A is impending to move towards right.

Since the block A is under equilibrium , ∑F Y = 0

∴ N 3 − F S2 sin45 − N 2 cos45 = 0

∴ N 3 − 0.25N 2 × 0.7071 − N 2 × 7071 = 0

∴ N 3 − 0.8839N 2 = 0 ....(2)



Also ΣF X = 0

−5 − F S3 − F S2 cos45 + N 2 sin45 = 0

................(from 1)∴ −5 − 0.25N 3 − 0.25N 2 × 0.7071 + N 2 × 0.7071 = 0 ....(from 1)

∴ −0.25N 3 + 0.5303N 2 = 5 ....(3)

Solving (2) and (3) simultaneously, we get N 3 = 14.2876kN and N 2 = 16.1642kN ....(4)

Block B is impending to move down

Since the block B is under equilibrium , ΣF X = 0

∴ N 1 sin60 − F S1 cos60 + F S2 cos45 − N 2 sin45 = 0

∴ 0.866N 1 − 0.25N 1 × 0.5 + 0.25N 2 × 0.7071 − N 2 × 0.7071 = 0 ....(from 1)

∴ 0.866N 1 − 0.125N 1 + 0.1768 × 16.1642 − 16.1642 × 0.7071 = 0 ....(from 4)

∴ 0.741N 1 − 8.5719 = 0

N_1 = 11.4939 kN ....(5)

Also $\sum F_Y = 0$

∴ $-W + N_1 \cos60 + F_{S1} \sin60 + F_{S2} \sin45 + N_2 \cos45 = 0$

∴ $N_1 \times 0.5 + 0.25N_1 \times 0.866 + 0.25N_2 \times 0.7071 + N_2 \times 0.7071 = W$

.................(from 1)

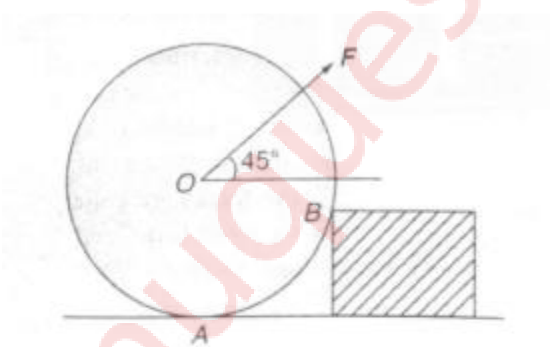
∴ $11.4939\times 0.5 + 0.2165 \times 11.4939 + 0.1768 \times 16.1642 + 16.1642 \times 0.7071 = W$.....(from 4 and 5)

∴ W = 22.5225kN

Hence a load of 22.5225kN can be held in the position. (ANS)

b. **(6)**

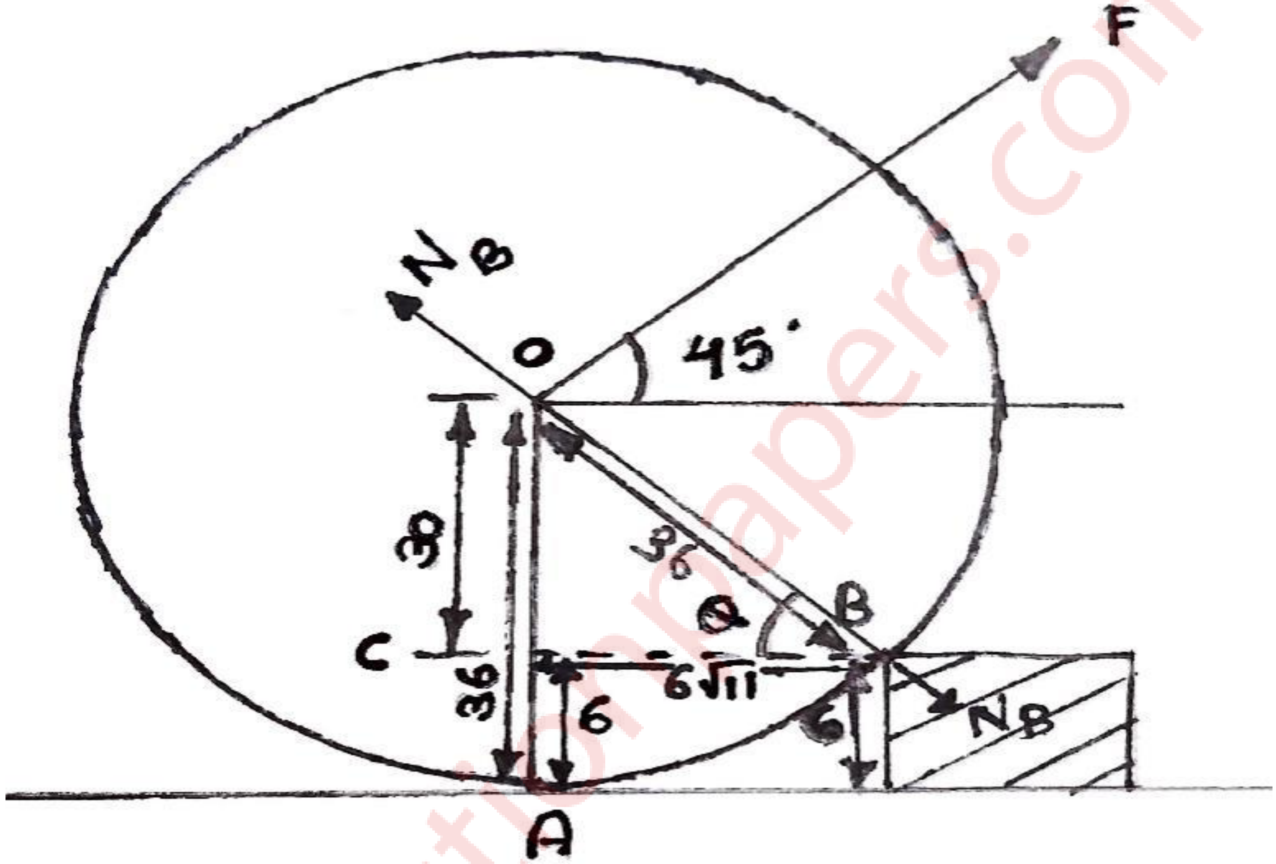
A road roller of radius 36cm and weighted 6000N, which is of cylindrical shape, is pulled by a force F, acting at an angle of 45° as shown in the figure below. It has to cross an obstacle of height 6cm. Calculate the force "F" required to just cross over the obstacle.



Soln:-

TO find the angle 'ϴ' some construction are done in the figure.

In ΔCOB, CB = √(36)^2 - (30)^2 = 6√11 cm



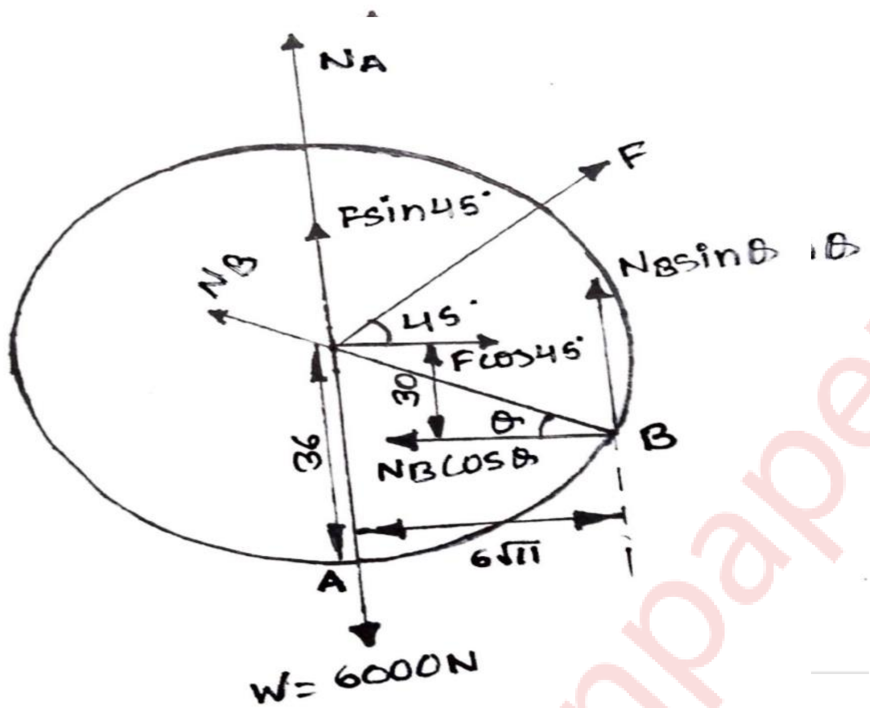
ϴ = sin-1(30/36) = 56.44°

N_B is the normal reaction between block and cylinder passing through center of cylinder.

FBD of cylinder:-

N_A is normal reaction of cylinder with ground.

Assume :- block of height 6 cm is of negligible mass.



$\Sigma F_x = F\cos(45°) - N_B\cos(56.44°) = 0$ .....(1) (taking right as +ve)

$\Sigma F_y = N_A - 6000 + F\sin(45°) + N_B\sin(56.44°) = 0$ (taking upwards as +ve)

$= N_A + F\sin(45°) + N_B\sin(56.44°) = 6000$ ....(2)

Moment of all forces about point B is zero.

Moment is equal to force * perpendicular distance of the line of force vector to the point.

All distance are in 'cm' convert them in 'm' .

$\Sigma M_B = 6000*(6\sqrt{11}/100) - N_A * (6\sqrt{11}/100) - F\sin(45°)(6\sqrt{11}/100) - F\cos(45°)*(30/100) + N_B*0 = 0$

$N_A * (6\sqrt{11}/100) + F\sin(45°)(6\sqrt{11}/100) + F\cos(45°)*(30/100) = 1193.98$ ....(3)

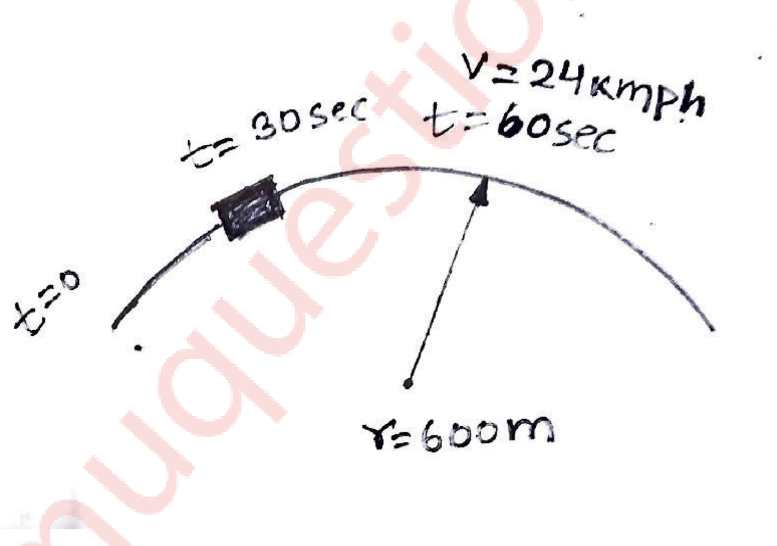
By eqauation 1, 2 and 3:-

F = 227.76 N (ANS)

c. **(6)**

At the instant t=0, a locomotive start to move with uniformly accelerated speed along a circular curve of radius r=600 m and acquires, at the end of the first 60 seconds of motion, a speed equal to 24kmph. Find the tangential and normal acceleration at the instant t=30 s.

Soln:-



At t = 30 sec :-

Normal acceleraltion $a_n = v^2/r$ (r = radius of curvature)

(v = 24kmph = 6.67m/s)

= 6.67/600

= 0.011m/s^2 (ANS)

Tangential acceleration a_t :-

$V = U + a_t*t$ (V=final velocity ,U=initial velocity,t=time taken)

$6.67 = 0 + a_t*30$

a_t = 0.22 m/s^2 (ANS)

4.)

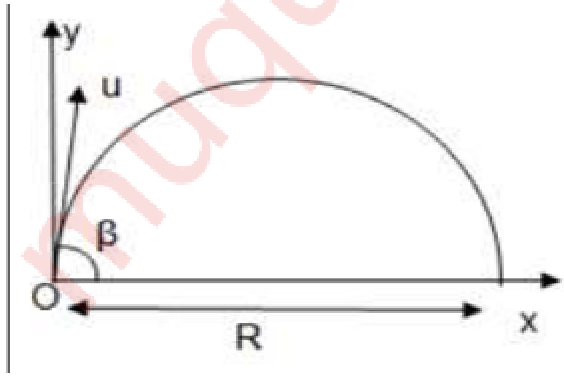
a. **(8)**

A particle is thrown with an initial velocity of 10 m/s at a 45° angle with horizontal. If another particle is thrown from the position at an angle 60° with the horizontal, find the velocity of the latter for the following situation:

(i) Both have the same range.

(ii) Both have the same time of flight.

Soln:-



Consider a particle performing projectile motion.

R – Horizontal Range

T – Total flight time

Considering vertical components of motion,

$s = ut + at 2$

$0 = u\sin(\beta)*T - gT^2$, $T = 2u\sin(\beta)/g$

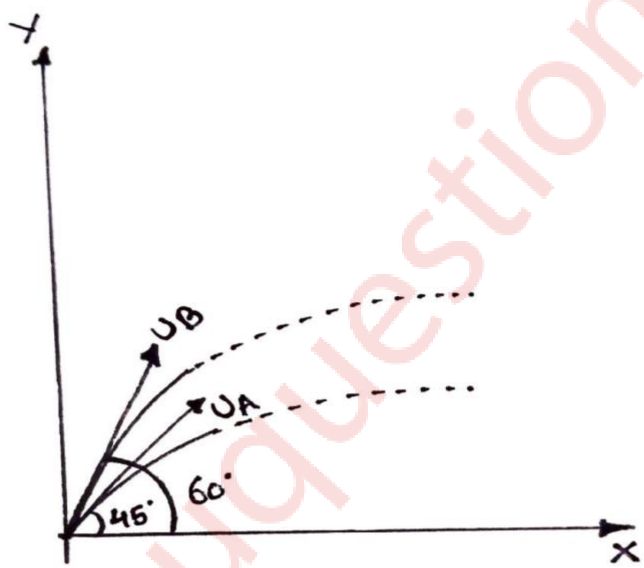
Considering horizontal components of motion,

$s = ut + at 2$

$R = u\cos(\beta)T + 0$ ....(as acceleration in x direction is zero)

$R = u\cos(\beta) \times 2u\sin(\beta)/g$

$R = (u^2\sin(2\beta))/g$



(I For same range :-

$(U_A)^2*\sin(2*45°))/g = (U_B)^2*\sin(2*60°))/g$

$10^2*\sin(90°)/g = (U_B)^2*\sin(120°)/g$

$U_B = 10.746$ m/s (ANS)

(II For same time of flight :-

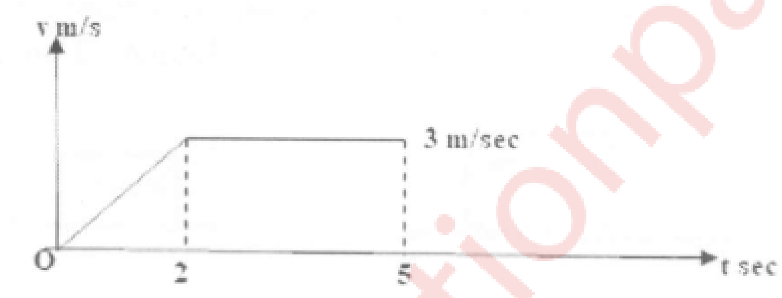
$2U_A\sin(45°)/g = 2U_B\sin(60°)/g$

$2*10*\sin(45°)/g = 2*U_B*\sin(60°)/g$

$U_B = 8.16$ m/s (ANS)

b. **(6)**

The motion of a particle is represented by the velocity-time diagram as shown in the graph shown below. Draw acceleration-time and displacement–time graphs.



Soln:-

(0-2)sec:-

Velocity is uniformly changing. So, acceleration will be contant and

a= (final velocity – initial velocity) / (final time – initial time)

= (3-0)/(2-0)

= 1.5m/s^2

For this time period curve of acceleration time graph will be 0° curve showing a constant value 1.5 m/s^2.

Displacement curve will be of 2° as velocity is uniform.

X2-X1 = area under velocity time graph for (0-2)sec

X2-0=½*3*2 , X2=3m

(2-5)sec:-

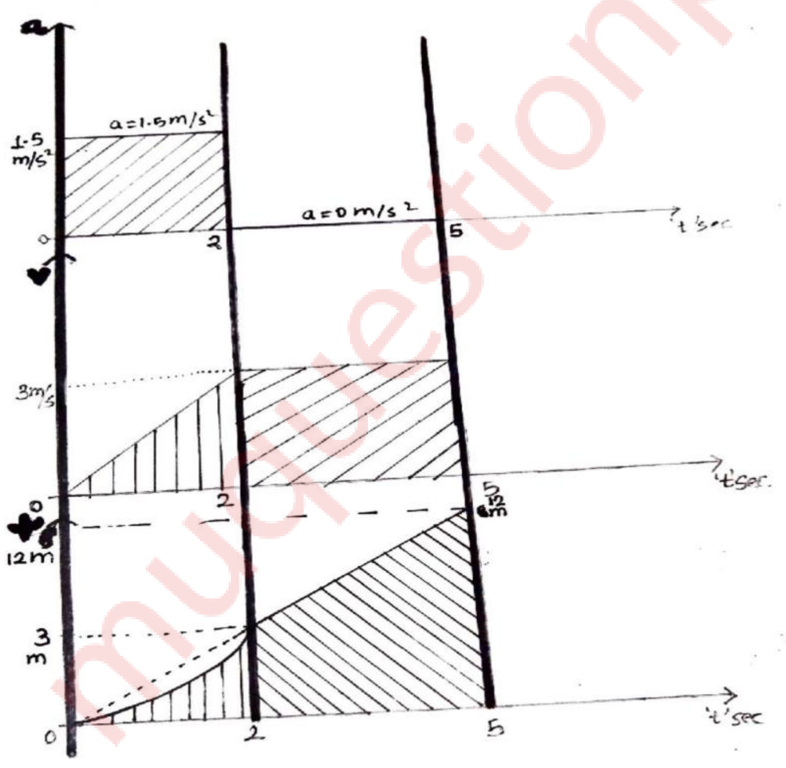
Velocity is constant for this time period.

So, acceleration time curve will be of 0° showing value 0m/s^2.

Displacement time graph curve will be of 1° as velocity is constant.

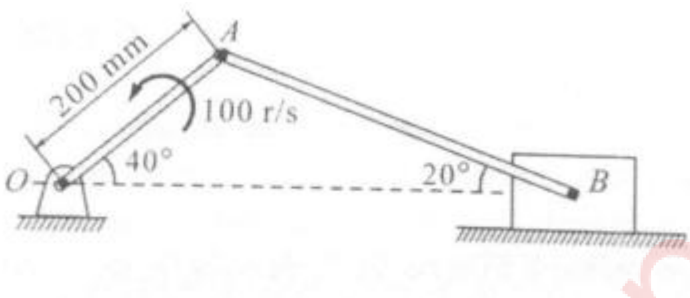
X5-X2 = area under velocity time graph for (2-5)sec

X5-3 = 3*3 , X5 = 12m



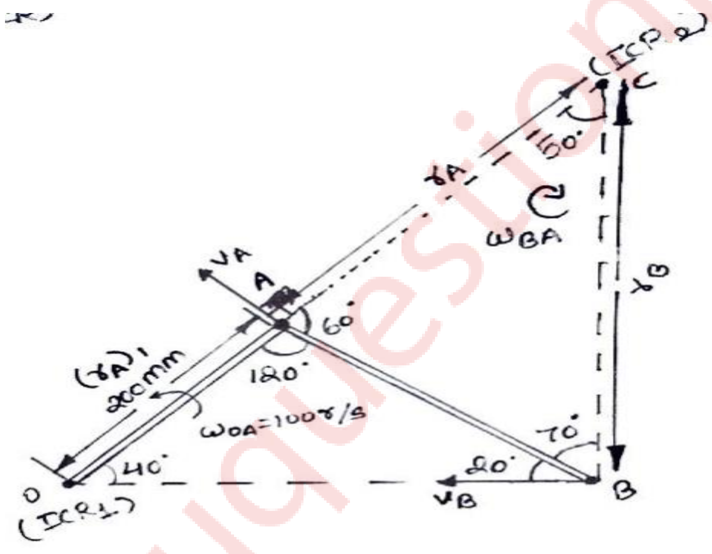
c. **(6)**

In the reciprocating engine mechanism shown in fig. The crank OA of the length 200mm rotates at 100 rad/sec. Determine the angular velocity of the connecting rod AB and the velocity of the piston at B.



Soln:-

Given:- 1. Motion of rod OA is considered for that ICR is point O and



velocity of point A is vA perpendicular to rod OA , ω_{OA} = 100rad/sec, radius of center of rotation of A is rA = 200mm or 0.2m(radius of center of rotation of point is measured from ICR).

2. Motion of rod AB from B to A , ICR of this motion c ,velocity of point A and B are vA nad vB, for this motion radius of center of rotation for A and B are rA and rB resp. And angular velocity is ω_{BA} = ?.

To find:- **ω_{BA} = ? , vB=?**

Calculation:-

1. Motion of rod OA:-

vA = ω_{OA}*rA

= 100*0.2

= 20m/s

2. Motion of rod AB:-

ω_{AB} = vA/rA = vB/rB

In ΔAOB,

By sine rule,

OA/sinB = AB/sinO = BO/sinA

(rA)1/sin20°=AB/sin40°

0.2/sin20°=AB/sin40°

AB=0.375m

In ΔABC,

By sine rule,

AB/sinC = BC/sinA = CA/sinB

0.375/sin50°= rB/sin60°=rA/sin70°

rA=0.460m

rB=0.424m

ωAB = vA/rA = vB/rB

ωAB = vA/rA

= 20/0.460

= 43.47r/s (ANS)

vB = **ωAB*rB**

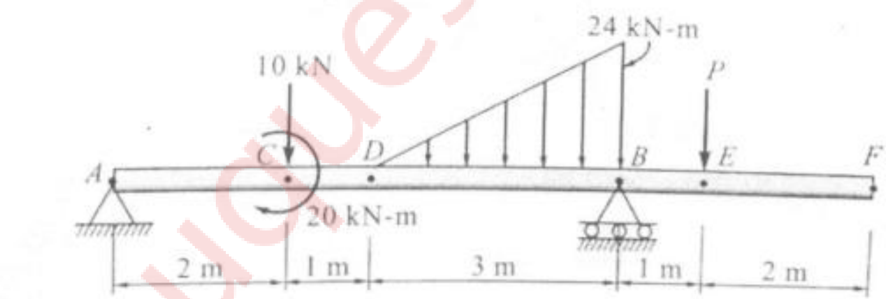
= 43.47*0.424

= 18.43 m/s (ANS)

5.)

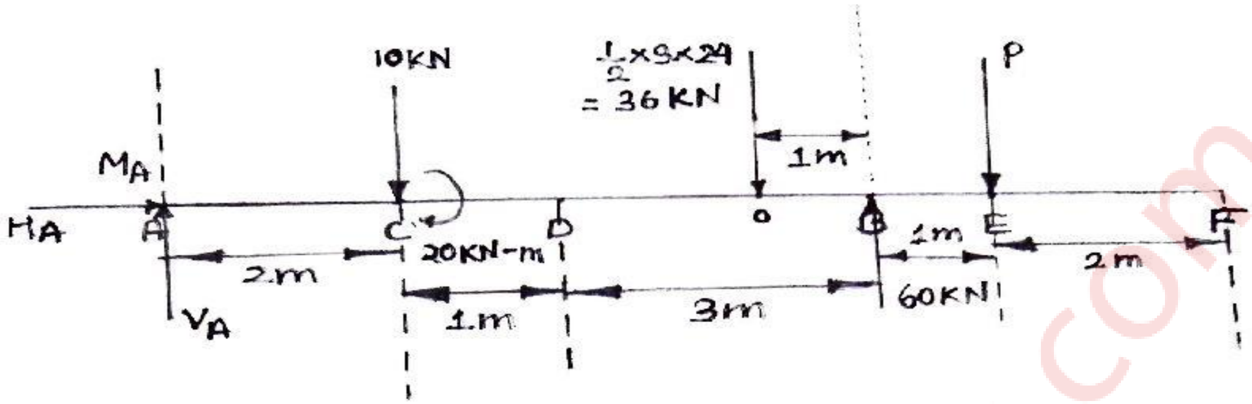
a. **(8)**

Find the support reaction at A and forces P if reaction at B is 60 kN for the beam loaded as shown in Figure below.



Soln:-

Given:- The figure 1st and 2nd .



∑Fx = 0

H_A = 0

∑Fy = -10 + 60 - 36 + V_A – P = 0

V_A – P = 14 ....(1)

∑M_A = -(Px7) +(60x6) – (10x2) -(36*5) = 0 (moment of forces along A is zero)

(taking anti-clockwise as +ve)

P = -22.85 kN or 22.85 kN (upwards) ....(2) (given direction of 'P' was wrong)

From (1) and (2)

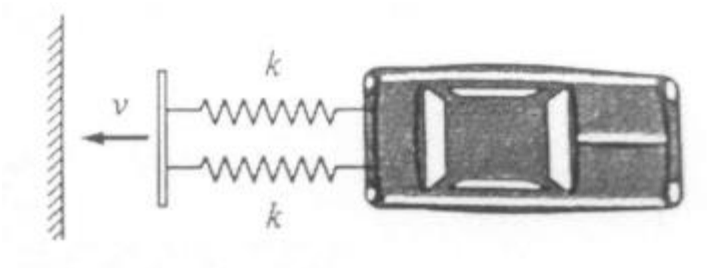
V_A = 36.85 kN

The magnitudes of force V_A , H_A = 0 and reaction P are 36.85kN and 22.85 kN respectively. (ANS)

b. **(6)**

A 1200 kg car has a light bumper supported horizontally by two springs of stiffness 15kN/m. Determine the initial speed of impact

with the fixed wall that causes 0.2 m compression. Neglect Friction.



Soln:-

Given :- Car of mass 'm' = 1200kg , spring of stiffness 'k' = 15*10^3N/m, initial and final compression are x1 and x2,initial and final speed are v1 and v2.

To find:- v1=?

Calculation:-

T1 = Initial kinetic energy = ½*m*(v1)^2 = ½*1200*(v1)^2 = 600*(v1)^2

T2 = Final kinetic energy = ½*m*(v2)^2 = ½*1200*0 (final velocity v2=0 as it bumps)

= 0

Work done by spring = ½*k_{eq}*((x1)^2-(x2)^2)   (resultant stiffness k_{eq})

Two springs of same stiffness are parallel.

So, resultant stiffness = 2k

x1 = 0m (no deflection) , x2= 0.2m

= ½*2k*(0 – (0.2)^2) = ½*2*(15*10^3)*(-(0.2)^2)   = -600J

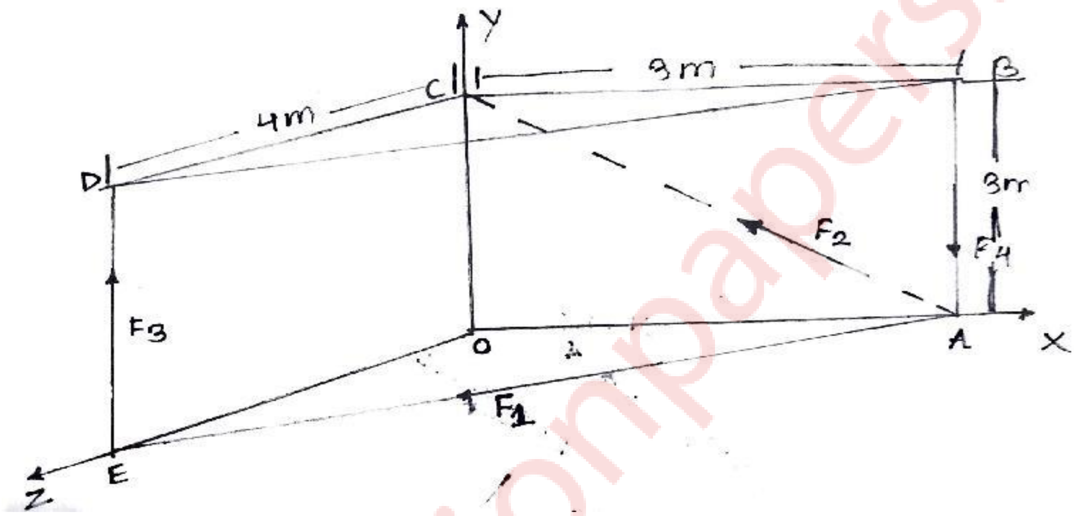
By work energy theorem:-

T1 + work done = T2

$600*(v1)^2 + (-600) = 0$

$v1 = 600/600 \quad = 1 m/s$ (ANS)

c. (6) Determine the resultant force of the force system shown in figure where F1=150N, F2=120N, F3=200N and F4=220N.



forces are given as:

$|\bar{F}1|=150N$, $|\bar{F}2|=120N$,$|\bar{F}3|=200N$, $|\bar{F}4|=220N$

WE CAN GET VELOCITY VECTOR BY MULTIPLYING THEM WITH UNIT VECTOR IN THEIR DIRECTION FROM THE DIAGRAM.

$\bar{F}1 = 150(3i + 4k)/\sqrt{3^2 + 4^2}$

$= 90i + 120k$

$\bar{F}2 = 120(3i + 3j)/\sqrt{3^2 + 3^2}$

$= 60\sqrt{2}i + 60\sqrt{2}j$

$\bar{F}3 = 200(3j + 4k)/\sqrt{3^2 + 4^2}$

$= 120j + 160k$

$\bar{F}4 = 220(3i - 3j)/\sqrt{3^2 + 3^2}$

$= 110\sqrt{2}i - 110\sqrt{2}j$

$\bar{R} = \Sigma\bar{F}$

$= (R_X)i + (R_Y)j + (R_Z)k$

$= \Sigma F_X + \Sigma F_Y + \Sigma F_Z$

$= (90+170\sqrt{2})i + (120-50\sqrt{2})j + (280)k$

$|\bar{R}| = \sqrt{(R_X)^2 + (R_Y)^2 + (R_Z)^2}$

$= \sqrt{(90+170\sqrt{2})^2 + (120-50\sqrt{2})^2 + (280)^2}$

$= 435.894$ N

$\cos\theta_x = R_X/R = (90+170\sqrt{2})/435.894 = 0.758$ (direction)

$\cos\theta_y = R_Y/R = (120-50\sqrt{2})/435.894 = 0.113$ (direction)

$\cos\theta_z = R_Z/R = (280)/435.894 = 0.642$ (direction)

$\Theta_x = 40.71°$

$\Theta_y = 83.51°$

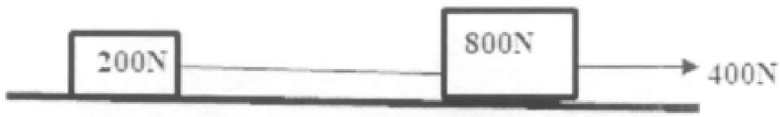
$\Theta_z = 50.05°$

6.)

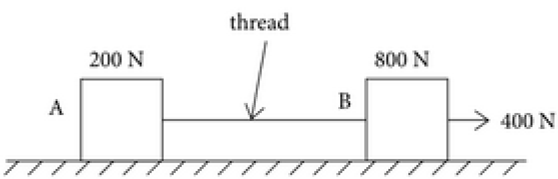
a. **(8)**

Two bodies A and B are connected by a thread and move along a rough horizontal plane (μ=0.3) under the action of 400N force

applied to the body as shown in Fig. Determine the acceleration of the two bodies and the tension in the thread using D' Alembert's principle.



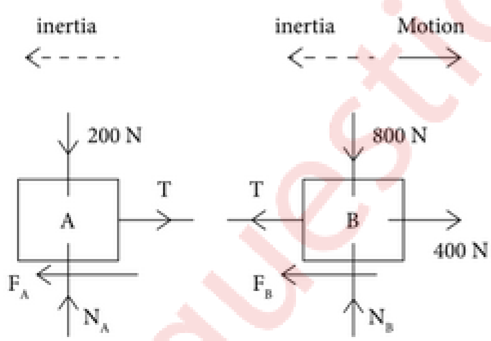
Soln:-



Given:- u=0 and μ = 0.3 .

The free body diagram of A and B is as shown below. Let blocks A and B are

accelerated by acceleration a_A and a_B respectively.



As both the bodies move to right side, their inertia will act opposite to motion as shown. Using D'Alembert's principle for body B,

Net force causing motion + inertia of body = 0

Net force causing motion = 400 – F_B - T

Inertia force of body = $(-800*a_B/g)$

$400 – F_B - T -800*a_B/g = 0$ .....(1)

And $N_B = 800$ N

Kinetics of particle:-

Thus equation (1) may be written as

$400 – \mu N_B - T = 800*a_B/g$

$400 – 0.3*800 - T = 800*a_B/g$

$160 - T = 800*a_B/g$ ......(2)

Similarly , Using D'Alembert's principle for body B,

$T – F_A – 200*a_A/g = 0$

$T – 0.3N_A = 200*a_A/g$

and, $N_A = 200$ N

$T – 60 = 200*a_A/g$ ....(3)

By putting $a_A = a_B = a$ and solving equation (2) and (3) :-

$160 - T +T– 60 = 200*a/g + 800*a/g$

$100 = 1000*a/g$ (g=9.81m/s^2)

a = 0.981 m/s^2

Substituting the value of 'a' in equation (3)

$T – 60 = 200*0.981/g$ (g=9.81m/s^2)

T = 80N (ANS)

$a_A = a_B = a = 0.981 m/s^2$ (ANS)

b. **(6)**

Train A starts with uniform acceleration of 0.5m/s^2 and attains a speed of 90km/hr which subsequently remains constant. One minute after it starts, another train B starts on a parallel track with a uniform acceleration of 0.9m/s^2 and attains a speed of 120km/hr. How much time does train B take to overtake train A.

Soln:-

TRAIN A:-

Initial velocity at A ‘uA’ = 90km/hr = 25m/s

Accleration of A ‘aA’ = 0.5m/s^2

Time taken ‘tA’= t sec

Distance covered = ‘sA’ m

TRAIN B:-

Initial velocity at B ‘uB’ = 120km/hr = 33.33m/s

Accleration of B ‘aB’ = 0.5m/s^2

Time taken ‘tB’= t-60 sec (Because train ‘B’ is starting 1 min late then Train ‘A’)

Distance covered = ‘sB’ m

Time of overtake will come when distance covered by both the trains are same.

Hence,

sA = sB

(uA)*(tA) + ½*(aA)*(tA)^2 = (uB)*(tB) + ½*(aB)*(tB)^2

25*t + ½*0.5*t = 33.33*(t-60) + ½*0.9*(t-60)^2

25*t + 0.25*t^2 = 33.33*t – 1999.8 + 0.45*(t^2 – 120*t + 3600)

0.20*t^2 – 45.67*t – 379.8 = 0

After solving the quadratic equation:-

t= 236.38 sec or –8.03 sec

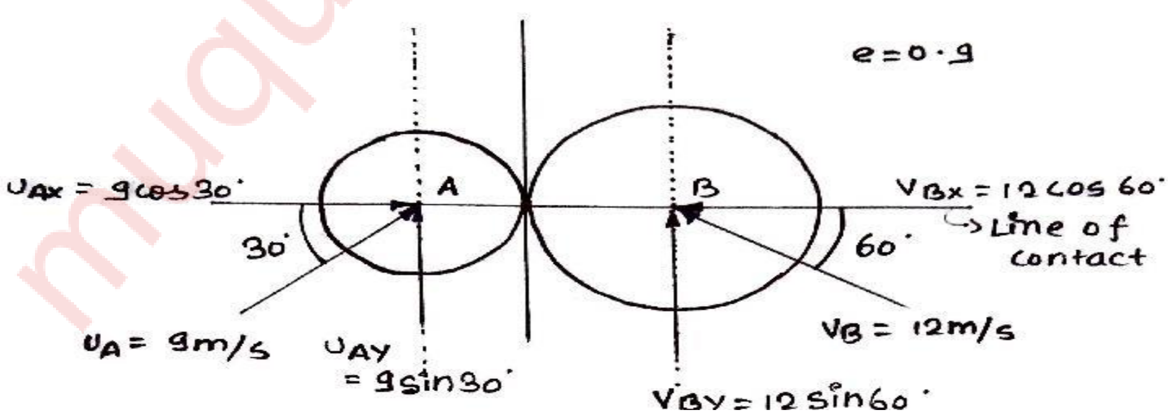
But time can't be negative so

T = 236.38 sec or 4min 56sec (ANS)

c. **(6)**

The magnitude and direction of the velocities of two identical spheres having frictionless surfaces are shown in figure below. Assuming coefficient of restitution as 0.90, determine the magnitude and direction of the velocity of each sphere after the impact. Also find the loss in kinetic energy

SOLUTION :-



Let mass of both identical bodies = m kg

Coefficient of restitution 'e' = 0.90

This impact is oblique collision.

U_{AX} = 7.794m/s (rightwards) U_{BX} = 6m/s (leftwards)

U_{AY} = 4.5m/s U_{BY} = 10.39m/s

$U_{AY} = V_{AY}$= 4.5m/s (UPWARDS)

$U_{BY} = V_{BY}$= 10.39m/s (UPWARDS)

By LCM :-

IM = FM

$m(7.794) + m(6) = m(V_{AX}) + m(V_{BX})$

$(V_{AX}) + (V_{BX}) = 13.794$ ....(1)

$e= (V_{BX} - V_{AX}) /(U_{AX} - U_{BX})$

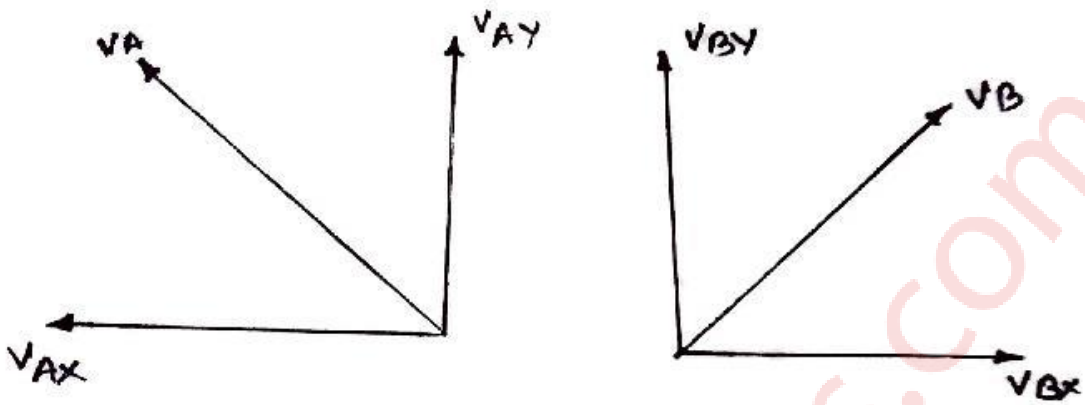
$V_{BX} - V_{AX} = 0.90*(7.794-(-6))$

$= 12.41$ ....(2)

By solving equation (1) and (2)

V_{BX} = 13.102m/s (rightwards)

V_{AX} = 0.692m/s (leftwards)



$V_A = \sqrt{(V_{AX})^2 + (V_{AY})^2} = \sqrt{(4.5)^2 + (0.692)^2}$

$= 4.55$ m/s (ANS)

$\Theta_A = \tan^{-1}(V_{AY} / V_{AX})$

$= 81.25°$ (ANS)

$V_B = \sqrt{(V_{BX})^2 + (V_{BY})^2} = \sqrt{(10.39)^2 + (13.102)^2}$

$= 16.72$ m/s (ANS)

$\Theta_B = \tan^{-1}(V_{BY} / V_{BX})$

$= 38.414°$ (ANS)

Loss in kinetic energy

$= \frac{1}{2}*(m1*m2/(m1+m2))*(1-e^2)*(u1\cos\alpha1 - u2\cos\alpha2)^2$

$= \frac{1}{2}*(m*m/(2m))*(1-(0.9)^2)*(9\cos(30°)-12\cos(60°))^2$

= **0.085 J (ANS)**